TACHEOMETRICAL SURVEYING

HANDBOOK ON

TACHEOMETRICAL SURVEYING

BY C. XYDIS

C.E. OF THE E.P.C. OF PARIS

London

E. & F. N. SPON, LIMITED, 57 HAYMARKET

New York

SPON & CHAMBERLAIN, 123 LIBERTY STREET

1909

PREFACE

THE present book is the result of the author's twelve years' personal experience of Tacheometrical Surveying. He hopes it will be of use to those about to take up this branch of the subject.

Tacheometrical Surveying is generally employed for the preliminary plans required in the erection of projected public works. These plans can be considered correct, as long as all measurements are taken by scale.

A trial to employ the tacheometrical method for the survey of estates and of wide portions of country has been made by the author, and the result being very satisfactory, he ventures to publish the method he employed in Chapter V.

The Tacheometrical Surveying Tables given at the end of this pamphlet will be found of practical use for the reduction of the tacheometrical book.

C. XYDIS.

ALEXANDRIA:
September 1, 1908.

CONTENTS.

HANDBOOK

ON

TACHEOMETRICAL SURVEYING

CHAPTER I.

OBJECT OF TACHEOMETRY.

THE object of tacheometry is to survey a plan with contours.

Tacheometrical Surveying is especially employed on sloping ground. In this case the ordinary method of surveying and contouring—viz. taking cross sections to a base line—is slow, whilst the method we propose to describe, is much more expeditious and nearly as accurate.

Tacheometer.—An ordinary transit theodolite with strong telescope fitted with stadia may be used as a tacheometer.

With a tacheometer we can get—

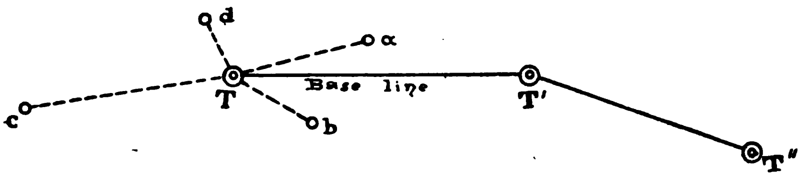


FIG. 1.

The angles of the points *a*, *b*, *c*, *d*, etc., with the base line.

The distances T*a*, T*b*, T*c*, T*d*, etc., with the stadia, and the differences of levels between T and *a*, *b*, *c*, *d*, etc.

Knowing the reduced level of T, we can calculate the reduced levels of *a*, *b*, *c*, *d*, etc.

We can in this way fill a portion of our plan with levels, and draw the contours according to the usual methods.

Stadia.—The object of the stadia is to measure distances without chaining.

The diaphragm of the telescope is fitted with two horizontal wires at an equal distance from the horizontal mean wire (see Appendix II., Elementary Theory of Stadia).

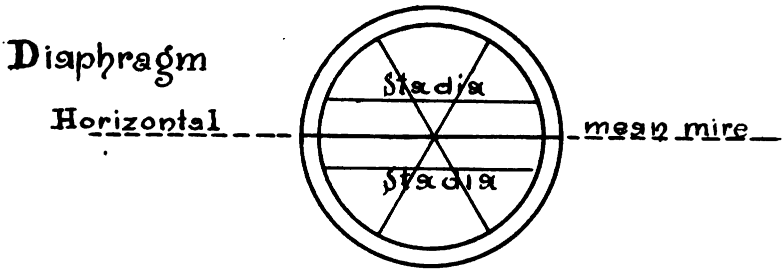


FIG. 2.

From the readings of these extreme wires to a staff (an ordinary levelling staff may be used) we can calculate the distance between the staff and the instrument peg.

The instrument makers can regulate the distance of the

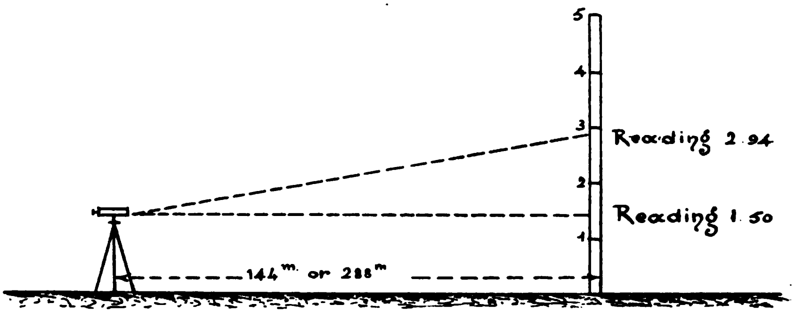


FIG. 3.

wires so that, for instance, 100 centimetres or 1 metre of difference of readings correspond to a distance of 100 m.

The scale of the stadia is then 1 : 100.

If we read 50 centimetres for 100 metres distance, the scale of the stadia is 1 : 200.

Example.—

Reading of the superior wire	294
„ „ inferior „	150
difference of readings	144

If the scale of the stadia is 1 : 200, the distance AB is equal to 288 m.

Remark No. 1.—To find the scale of a stadia we measure 100 metres on flat ground, we adjust the tacheometer and note the readings of the extreme wires. If the difference of readings is 1 metre, the scale is 1 : 100.

The scale will be 1 : 200, if the difference is 0 m. 50.

Remark No. 2.—The reading of the mean wire is equal to the sum of the reading of the extreme wires of the stadia divided by 2. In the preceding example the reading of the mean wire is $\frac{294 + 150}{2} = 2$ m. 22.

Remark No. 3.—When reading with the stadia, it is better to clamp the inferior wire to a round figure if possible, viz. 1 m. 00; 2 m. 00. The subtraction in this case is easier.

Remark No. 4.—The ground is called flat, when the declivities or acclivities do not exceed 3°–4°.

Use of the Stadia on Steep Ground.—

T the telescope of the instrument (Fig. 4).

A B the position of the staff at right angles to the direction of the telescope.

A C the vertical position of the staff.

We can always suppose that the direction of the telescope is parallel to the slope.

Z the zenithal angle.

d the horizontal distance between the instrument and the staff.

k the scale of the stadia.

s_1 the reading at position A B perpendicular to the slope.

s_2 „ „ „ A C vertical „ „

x_1 the distance measured on the slope.

We have

$$d = x_1 \sin Z$$

$$x_1 = k\, s_1$$

consequently

$$d = k\, s_1 \sin Z$$

but

$$s_1 = s_2 \sin Z$$

therefore

$$d = k\, s_2 \sin^2 Z \quad . \quad . \quad . \quad (1)$$

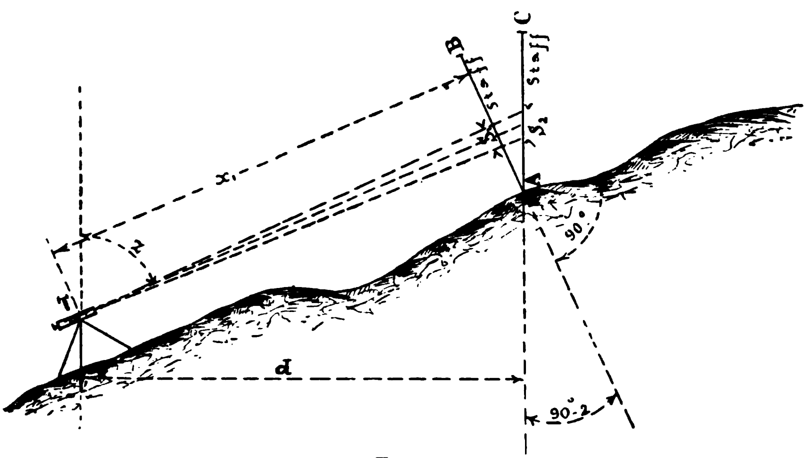


FIG. 4.

Rule No. 1.—To find the distance between the instrument and the staff multiply the difference of readings by the square of the sine of the zenithal angle, and by the scale of the stadia.

Example	scale of stadia . . .	100
	superior wire . . .	280
	inferior	100
	difference	180
	zenithal angle . . .	80°

$\sin 80° = 0{\cdot}9848$

$\sin^2 80° = 0{\cdot}9698$

Correct distance 180 × 0·9698 = 174 m. 56.

If the zenithal angle is 87° the corrected distance will be

$$180 \times 0{\cdot}9972 = 179 \text{ m. } 50$$

or 0 m. 50 difference between 180 m.

In tacheometry we cannot vouch for an accuracy of more than 0 m. 50.

Rule No. 2.—The correction of $\sin^2$ may be neglected when the zenithal angle is between 87° and 93°.

Note.—We supposed in the above that the direction of the telescope is parallel to the slope. In practising with the tacheometer we must always see that this condition is practically fulfilled.

CHAPTER II.

TACHEOMETRICAL LEVELLING.

Problem No. 1.—Find the difference of levels between the instrument peg and the foot of the staff.

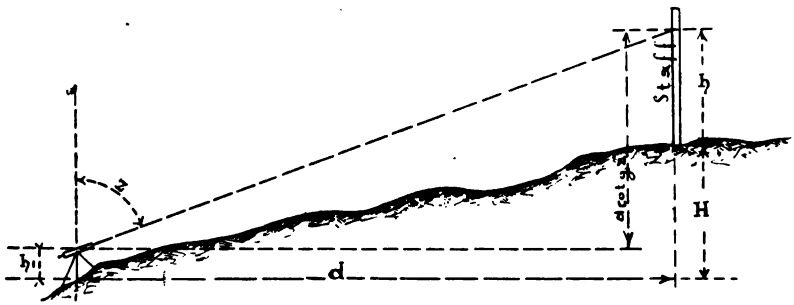


FIG. 5.

Z the zenithal angle.
h_1 the height of telescope from the station peg.
h the reading of the mean wire or sum of the readings of the extreme wires divided by two.
d distance between station peg and foot of the staff.
H the difference of level.

We have:

$$H + h = h_1 + d \cot Z \quad . \quad . \quad (2)$$

therefore

$$H = h_1 + d \cot Z - h \quad . \quad . \quad (3)$$

In the sketch the staff is above the instrument, but we may use the same formula when the staff is below the instrument.

h is always negative
$d \cot Z$ is positive when Z is less than 90°, and negative when Z is more than 90°.

We can also write

$$d \cot Z = k\, s_2 \sin^2 Z \cot Z = k\, s_2 \sin^2 Z \frac{\cos Z}{\sin Z}$$
$$= k\, s_2 \sin Z \cos Z$$
$$= k\, s_2 \sin \frac{2 Z}{2}$$

Example :—

Readings of extreme wires $\frac{324}{100}$ or 224 distance to be corrected ; mean wire $\frac{324 + 100}{2}$ = 2 m. 12.

zenithal angle 86°.
h_1 = 1 m. 45.

Find the correct distance and the difference of levels.
Correct distance

$$d = 224 \times \sin^2 86° = 224 \times 0{\cdot}9975^2$$
$$d = 224 \times \quad = \quad 222 \text{ m. } 92$$

difference of level formula (3)

$$H = 1 \text{ m. } 45 + 222{\cdot}92 \times \cot 86° - 2 \text{ m. } 12$$
$$H = 1 \text{ m. } 45 + 222{\cdot}92 \text{ m.} \times 0{\cdot}0699 - 2 \text{ m. } 12$$
$$H = 15{\cdot}59 - 2 \text{ m. } 12 + 1 \text{ m. } 45$$

Thus

$$H = 14 \text{ m. } 92.$$

Problem No. 2.—Knowing the reduced level of the instrument peg, find the reduced letter of the foot of the staff.

R reduced level of instrument peg
R_1 „ „ the centre of the telescope
x the unknown reduced level of the foot of the staff.
h_1 height of the telescope above the ground
H the difference of levels (Fig. 5).

We have

$$x = R + H$$
$$R_1 = R + h_1$$

H may be positive or negative according to the position of the staff.

But

$$H = h_1 + d \cot Z - h$$

consequently

$$x = R + h_1 + d \cot Z - h$$

if we make

$$d \cot Z - h = A \quad . \quad . \quad . \quad (4)$$

A may be positive or negative

therefore

$$x = R_1 + A \quad . \quad . \quad . \quad (5)$$

Rule No. 3.—To calculate the reduced level of the foot of the staff we add to the reduced level *of the telescope* the quantity A or $d \cot Z - h$ with its sign + or −.

Recapitulation.—Referring to Fig. 1.

We can, by sending the staff to *a, b, c, d,* etc.—

1. Measure the horizontal angles of *a, b, c, d* either with the base line or with the magnetic north.

2. Calculate from the readings of the wires the distances T*a*, T*b*, T*c*, T*d*, etc. (Fig. 1), according to rules 1 and 2.

3. Calculate from the reading of the zenithal angle Z and the mean wire, the reduced levels of the foot of the staff according to rule 3.

All those points *a, b, c, d,* etc., may be plotted and the contours drawn as usual.

In the following chapters we will explain how this work is carried out on the field and in the office, but first we will say a few words on the accuracy of the tacheometrical method.

Accuracy of the Tacheometrical Method.

Suppose that the base line is 0 − 1 − 2 − 3 − 4, etc.

In order to measure, the distance 0 − 1 we plant the instrument above peg 0 and send the staff-holder above peg 1, at the same time we measure the zenithal angle, and the magnetic bearing 0 − 1.

When the station 0 is finished we fix the instrument at 1

and send the staff-holder to peg o. We measure the bearing 1 – o, the distance 1 – o, and the zenithal angle, thus :—

The magnetic bearing is measured twice (with a difference of 180° + the variation of the compass), the distance o – 1

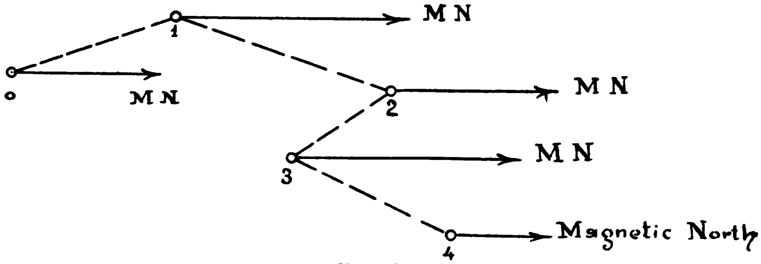


FIG. 6.

twice, and the difference of levels also twice. Proceeding in this manner we can see on the spot if there is any mistake, in which case we must start the operation again.

According to our own experience we must not rely on an accuracy of more than 0 m. 50 to 1 m. for distances of 300 m. –. 350 m. and 4 – 5 centimetres for difference of levels of 10 m. between station pegs. Supposing that our base line is composed of 50 sides of 350 m. average length, what will be the probable error of length at the end of the base line, the accuracy of distance being 1 m.

$$\begin{aligned}\text{The probable error is } &= \pm 1 \text{ m.} \sqrt{50} \\ &= \pm 7 \text{ m. nearly}\end{aligned}$$

Thus our base line may be shorter or longer by 7 m. in a total length of 350 × 50 = 17,500 m.

The possible error, then, is

$$\frac{7}{17500} = 0{\cdot}0004.$$

Generally this error may be considered as unimportant.

The probable error for levelling after 50 stations is

$$\begin{aligned}\text{Probable error } &= \pm 5 \sqrt{50} \\ &= \pm 0 \text{ m. } 35.\end{aligned}$$

For a reconnaissance plan this error of 0 m. 35 cm. is not important, but for a plan at a scale of 1 : 1000 or even 1 : 2000 it is important, as all the contours may be 35 cm. wrong.

The best thing in this case is to level from the nearest bench-mark, to one of the last station pegs. In case there are no bench-marks near the base line, a longitudinal levelling of all the station pegs is necessary.

CONCLUSION.

After adjusting the tacheometer as an ordinary transit theodolite, the necessary precautions to take to have an accurate tacheometrical survey are—

1. Test your stadia, and if the telescope is fitted with a Porro's lens (see Appendix), correct the measuring angle, if necessary, with the key.

2. The readings of the two verniers of the vertical circle must be 90° – 270° or 0° – 180°, when the line of collimation is truly horizontal; if this is not the case, correct it as you would for a transit theodolite.

3. We draw the attention of the surveyor to the correct adjustment of the parallax; this is of great importance when working with stadia, as any neglect will cause an error of 2–3 metres.

4. Take the horizontal angles forwards and backwards with the magnetic north (in non-magnetic ground you should not have a difference of more than 10′–20′).

5. Measure your distances forwards and backwards and take the mean distance when the difference is small.

6. Calculate the difference of levels between two station pegs on the spot, and take the mean level when there is a small difference. In case of an important difference start the readings again.

7. Direct the telescope practically parallel to the slope of the ground.

8. Do not take zenithal angles of less than 60°.

9. Every 10 or 12 stations try to find a bench-mark and see if there is an important difference in levelling.

10. In case of an important tacheometrical survey, when you cannot find bench-marks near the station-pegs, a longitudinal levelling of the station-pegs should be carried out.

11. Take the angles from different stations of all the important objects you may see, such as chimneys, church-steeples, etc.

12. Try to work in good atmospherical conditions, avoid windy and foggy days, and stop the work when the sun heats the ground and causes the air to vibrate.

13. Try to obtain intelligent men as staff-holders. The quantity of work done per day depends not only on the skill of the surveyor but also on the training of the staff-holders.

14. The staff must always be perfectly plumb, as in ordinary levelling.

CHAPTER III.

TACHEOMETRICAL BOOK-KEEPING.

WE give in Fig. 7 of this book an example of tacheometrical book-keeping.

The tacheometrical book is divided into sixteen columns.

The first six columns are for field work and the other ten are calculated in the office.

Column No. 1 is for the number of station pegs.
- „ No. 2 for the height of the telescope above the peg (h_1 formula 3).
- „ No. 3 for the number of staffs.
- „ No. 4 for the horizontal angle.
- „ No. 5 for the zenithal angle.
- „ No. 6 for the readings of the extreme wires.
- „ No. 7 for the difference of extreme wires (s_2 formula 1).
- „ Nos. 8 and 9 for h, or height of the mean wire above the ground according to the divisions of the staff, or in metres.
- „ No. 10 for $d \sin^2 Z$ or corrected distance.
- „ No. 11 for $d \cot Z$.
- „ No. 12 for $d \cot Z - h$ or A when positive.
- „ No. 13 „ „ „ negative.
- „ No. 14 for the reduced level of the telescope R_1 (formula 5).
- „ No. 15 for reduced level of station pegs and points.
- „ No. 16 for remarks.

Field Book.—The surveyor should be provided with an ordinary field book in order to make a neat sketch map, showing streams, roads, paths, buildings, etc.

Station Pegs	Height of Instrument	No. of Points	Angles		Wires	Difference of Extreme Wires s_2	Height of Mean Wire		Correct Distance	d cot Z	d cot Z − h or A		Reduced Level		Remarks
			Horiz.	Zenit.			h	h			+	−	Of Instru-ment	Of Pegs	
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	1·49	B	20°	83°·31	$\frac{355}{100}$	255	2·27	2·27	255	6·60	+6·60+	1·49−	2·27=	+5·82	5·85
B	1·52	A	199°·40	91°·22	$\frac{305}{50}$	255	1·77	1·77	355	5·63	−5·63−	1·77+	1·52=	−5·89	

FIG. 7.

CHAPTER IV.

TACHEOMETRICAL TRAVERSING FIELD WORK.

WE will now describe tacheometrical surveying in the field. The surveying party consists of:—

A surveyor in charge of the instrument.
An engineer in charge of the field book.
An assistant in charge of the book-keeping.
6 or 8 staff-men.
1 or 2 porters to carry the instrument, umbrella, etc.

We have to be provided with the following articles:—

1 tacheometer.
1 umbrella with long stick.
6 or 8 staffs.
1 tape to measure the height of the telescope.
1 slide rule or tacheometrical tables.
1 tacheometer book.
1 field book.
3 whistles.
1 field glass.
1 plumb-bob.
Nautical almanack of the year.

Suppose we have to start from station A of Fig. 8.

The surveyor plants the tacheometer above peg A and adjusts it.

The book-keeper measures the height of the telescope and writes it in column 2.

The engineer in the meantime makes a reconnaissance of the proposed survey, and chooses the place of station peg B.

He makes a neat sketch *of all the features* such as roads,

paths, streams, etc., and puts the staff-men in position for the first readings, as in Fig. 8.

He puts a peg and a rod in station B.

The surveyor reads the magnetic bearing A B, and starts the reading of the staffs, calling out to the book-keeper: Extreme wires, for instance, 324–100; zenithal angle 85° 35′; horizontal angle, 185° 24′. The book-keeper when taking them down must repeat them aloud, in order to avoid mistakes.

When the reading of one staff is finished, the staff-holder goes at once to his new position, so that when the readings of

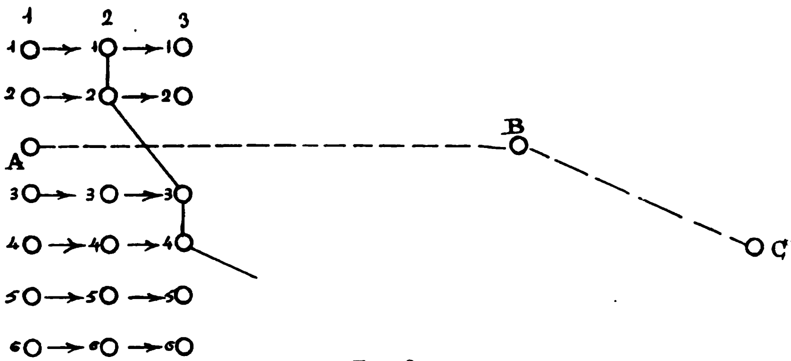


FIG. 8.

staff 6 are finished, the surveyor can start immediately the readings at staffs 7 and 8, thus avoiding a waste of time. The engineer has to see to all these things, and show the staff-men their proper places.

When all the necessary points from station A are finished, one of the staff-men must go to station B, and the surveyor reads the distance, the vertical angle and the horizontal angle. In this way he can see that his instrument did not shake in the meantime.

The readings of station B being finished, the surveyor gives instructions to carry the instrument, and goes on to station B. During the whole operation the leader must stand between the instrument and the staff-men, in order to inspect

the positions of the staffs, and also to see that the numbers in his field book agree with the numbers in the tacheometrical book.

At station B the surveyor adjusts his instrument. In the meantime the leader sends a staff-man to station A, and goes himself to reconnoitre the ground towards station C, choosing the place of station C and placing a peg and rod. The surveyor reads the bearing B—A, the distance and the zenithal angle.

The difference between two magnetic bearings must not exceed 10–20 minutes (except in ferruginous ground).

The corrected readings between A B and B A must be equal, and the difference of level calculated on the spot must not exceed 5–6 centimetres. This calculation is done with a slide rule or with tables according to formula 3 (see Fig. 7).

These three checkings of horizontal bearing, distance and difference of level are very important, and we particularly draw the attention of tacheometrical surveyors to this subject. When these checkings are finished, the surveyor can start the reading of bearing B C and the reading of the points from station B, as previously described in the case of station A. We conclude this chapter by saying a few words on the duties of each man belonging to the staff.

Engineer.—The engineer has to take a reconnaissance of the ground, choose the place of the station pegs and the position of the staffs, and see if the numbers in his field book correspond to those in the tacheometrical book. He has really the direction of the whole work, and all questions must be referred to him, so that the surveyor should not be disturbed from his readings. It is advisable for the engineer to place the staff-men in nearly a straight line perpendicular to A B, the most competent being at the two ends, as it is more difficult to communicate with them, besides which they can place the others in the straight line required. The placing of the staffs depends of course on the shape of the ground and in many cases we cannot act in the way we have described. The leader must exercise his own judgment and from his

personal experience, from the shape of the ground, from the object of the contour plan, decide on the most advantageous way of placing the staffs.

Surveyor.—The surveyor plants and adjusts his instrument. When the readings are finished he whistles to the staff-men to take up their next positions.

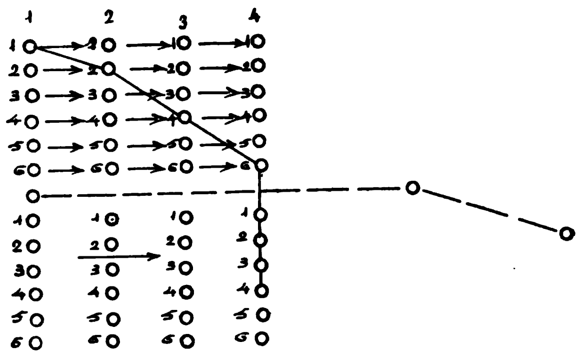


FIG. 9.

Book-Keeper.—The book-keeper stands near the instrument, measures the height of the telescope above the peg, and writes it in column 2, Fig .7. He has also to help the surveyor to calculate the difference of levels.

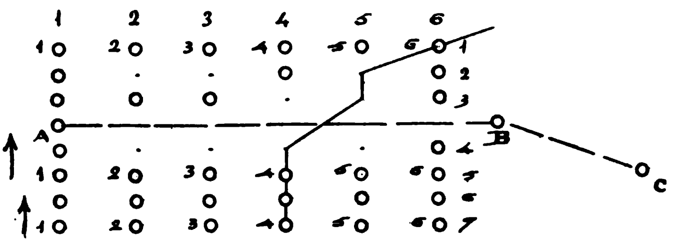


FIG. 10.

In Fig. 8, we suppose that the 6 staffs are enough to take a cross section. In case the leader finds that 6 points are not enough he can put 4 or even 6 on the one side of the base line, and afterwards start the surveying of the other side (see different ways of placing staff-men, Figs. 9 and 10).

Subsidiary Stations.—In case the width of plan required is more than 300–350 m. on each side of the base line, subsidiary stations must be employed like those in the sketch.

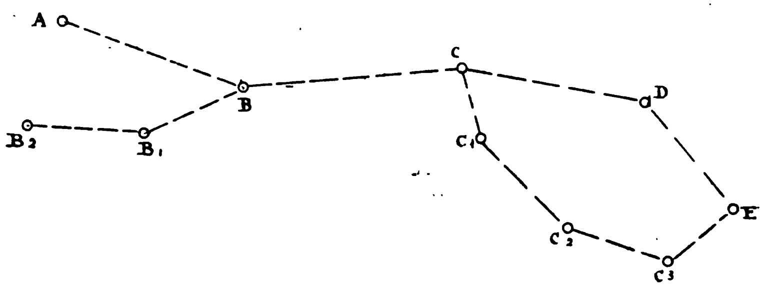


FIG. 11.

B_1 B_2 subsidiary stations.

C_1 C_2 C_3 subsidiary stations.

The line C C_1 C_2 C_3 E D is closed, and we can see if the angles are correct as the sum of angles at C, C_1, C_2, C_3, E and D must be equal to

$$(6-2) \times 180^\circ = 720^\circ.$$

The reduced levels of E through C D E, and through C, C_1, C_2, C_3, E must be equal.

Remark No. 1.—The surveyor must be careful about his readings, as a wrong reading altogether alters the shape of the contours. He need not read the horizontal angles of the points to the minute, but need only quote the 5 minutes. In this way he can read the horizontal angle quickly and without the help of the magnifying glass of the vernier.

The error is quite unimportant.

Example.—Suppose we read 35° 45′ instead of 35° 42′, which is the correct reading, and which makes a difference of 3 minutes. At a distance of 350 m. the difference will be:

$$350 \text{ m.} \times \tan 3' = 350 \times 0{\cdot}000873 = 0 \text{ m. } 31 \text{ cm.}$$

which at the scale of 1 : 1000 will be less than $\frac{1}{3}$ mm. and less than $\frac{1}{6}$ in the scale of 1 : 2000.

This difference cannot alter the shape of the contours.

Remark No. 2.—We suppose that our station pegs are at

a distance of 350–400 m. In order to reduce the number of stations and increase their distances we can put a subsidiary peg half-way between A and B in the direction of the line A B.

We can read the distances A A_1, A_1 A, A_1 B and B A_1.

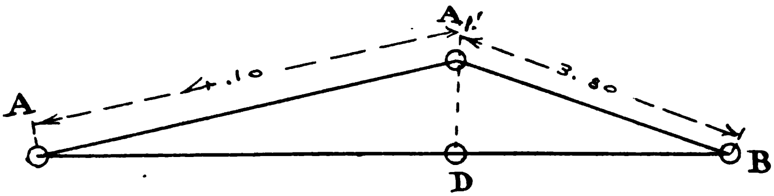


FIG. 12.

If the peg A_1 is in a straight line between A and B, we need not take any horizontal angles and put

$$A\,A_1 + A_1\,B = A\,B$$

The horizontal and zenithal angles have of course to be taken from station A to B and from B to A. In this manner we can double the distances between the station pegs.

Let us calculate the maximum error we can make in the most unfavourable case in taking A A_1 + A_1 B = A B.

Suppose we are 3 m. out of line A B.

We have the correct distance A A_1 = 410 m.
" " " A_1 B = 280 m.
We want the correct distance A B.

We have $\sin A_1 A B = \frac{3}{410} = 0{\cdot}0073$, and $\sin A_1 B A = \frac{3}{280} = 0{\cdot}0107$.

Therefore

$$\text{angle } A_1 A B = 25'$$
$$\text{,, } A_1 B A = 37'$$

$$A D = 410 \text{ m.} \times \cos 25' = 410 \times 0{\cdot}999974$$
$$D B = 280 \text{ m.} \times \cos 37' = 280 \times 0{\cdot}999942$$

A D	= 409 m. 99
D B	= 279 m. 98
A D + D B = AB	= 689 m. 97
690 − 689·97	= 0 m. 03

or 3 cm. difference, which is quite immaterial.

Average Work Per Day and Mistakes of Work.

In easy ground we can take in a fair day 700 points.

In steep or woody ground we can only take 300.

Supposing the points are at a distance of a chain and a half, if we take 12 points per cross section, we can survey in the first instance 1740 m. of base line per day and 750 m. in the second instance.

The surveyor has to take 700 × 3 = 2100 readings.

Some of these may be wrong, but with a neat sketch map, a great many of them may be corrected, as for instance, mistakes of 10° or 50° of horizontal angles. Mistakes of distances, too may be found out, if the staff-men are in a row, as shown in the sketch below.

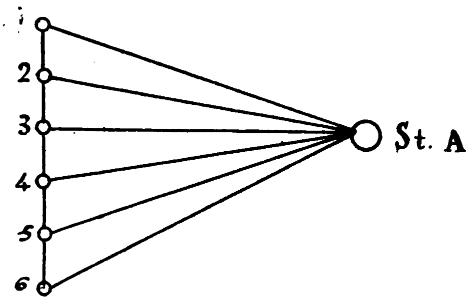


Fig. 13.

Suppose the distance of staff 3 is not correct. As the mistake is usually of 10 m., 20 m., 30 m., we can see what figure corresponds nearly to the straight line 1, 2, 3, 4, 5, 6.

The surveyor must be careful about his readings especially concerning zenithal angles.

OFFICE WORK.

Columns Nos. 1, 2, 3, 4, 5, 6 are filled in in the field.

The rest of the columns are calculated in the office.

Column No. 7 is the difference of the readings of extreme wires or non-corrected distances.

1	2	3	4	5	6	7	8	9
			Angles.				Height of Mean Wire	
Stations Pegs	Height of Instrument	No. of Points	Hor.	Zen.	Wires	d	in Staff	in Metres
Station B . . .	1·45	A						
		C						
		1	145·35	81° 0′	$\frac{380}{200}$	180	..	2·90
		2	156·00	82° 10′	$\frac{392}{200}$	192	..	2·46
		3	161·45	82° 43′	$\frac{355}{50}$	305	..	1·52
		4	164·20	84° 30′	$\frac{462}{100}$	362	..	1·81

FIG. 14.

Column No. 8 is for stadia 1 : 200 or 1 : 50.

We take scale of the stadia 1 : 100, so we have to write in column No. 9 the sum of the two readings of column No. 6 divided by two.

Column No. 10 is for corrected distances, or $d \sin^2 Z$.

In case of point 1 we have

$$180 \times \sin^2 81°.$$

Different ways of calculating $180 \times \sin^2 81°$.

1. By logarithms we have taking the log

$$\log 180 + 2 \log \sin 81° = 2{\cdot}2445$$
$$\log 175{\cdot}60 = 2{\cdot}2445$$

175·60 is the correct distance.

2. There are special tables calculated, for the different values of d from metre to metre up to 300 m. and for the different values of Z between 50° and 90°. Generally those tables are for a decimal division of degrees.

3. By graphical tables

Representing by polar co-ordinates the curve

$$R = d \sin^2 Z$$

This curve is a sort of spiral.

Giving to d different values say 10, 20, 30, etc., we have different parallel curves. This curve is to be calculated only for values of angle Z between 90° and 45°, steeper readings being unusual.

With an ordinary table of natural sines and cosines we calculate $d \sin Z$, and consequently $d \sin^2 Z$.

We plot the different angles from 0° to 45° and the values of $d \sin^2 Z$. Thus we can draw a graphical table like the one at the end of this book. The scale of length is 1 mil. per metre and the scale of angles 6° per 1°.

Example.—

Find $d \sin^2 Z$ for 180 and 81°.

Measuring o–a we find that the correct distance is between 175 and 176 and the maximum error may be o m. 50 (see Spiral for Reduction of Distances).

4. Further on, we will show how to calculate $d \sin^2 Z$ with a slide rule.

Calculation of d cot Z.

1. By tables as in the case of $d \sin^2 Z$.

2. By graphical tables. We have in this case to calculate the spiral

$$R = d \cot Z$$

giving to Z values from 0° to 30°, and to d 10 m., 20 m., 30 m., etc.

We have as many parallel spirals as we like.

3. By slide rule.

It is undoubtedly the quickest method of calculating $d \sin^2 Z$, and $d \cot Z$.

The theory of the slide rule is based on logarithms.

Suppose we have to calculate

$$y = d \cot Z.$$

Taking the logarithms we have

$$\log y = \log d + \log \cot Z.$$

We have only to add the logarithms and find the corresponding number in the Tables.

Then marking on a rule the logarithm of d and in the sliding part the logarithm of cot Z, we slip the origin of logarithm cot Z to the number d, and the number corresponding to Z is $d \cot Z$. In fact we make a graphical addition of $\log d$ and $\log \cot Z$.

There are several patterns of slide rules. We mostly use the Kern (Aarau-Switzerland) pattern, the length of which is about 1 foot, and gives by a single slide

$$d_1 = d \sin^2 Z \text{ and } d_1 \cot Z.$$

For reckoning up the book with the slide rule in the office, the book-keeper reads d in column No. 7, and the zenithal angle in column No. 5. The surveyor calculates with the rule those quantities which the book-keeper writes in columns Nos. 10 and 11. Column No. 12 is calculated easily. It is A or $d \cot Z - h$ when positive.

When $d \cot Z - h$ is negative, it is written in column No. 13. Column No. 14 is for the reduced level of the telescope.

R_1 – (formula 5) which is equal to the reduced level of the peg, plus the height of the instrument.

Column No. 15 is for the reduced levels of the points.

We have to add or subtract A or $d \cot Z - h$ from R_1 according to formula 5.

Remark 1.—The slide rule is always sufficient to calculate $d \cot Z$ for points, even when the distance is great and the

slope steep, but for station pegs 300–350 m. apart, and zenithal angles of less than 80°, we prefer the use of the tables to that of the slide rule, as there might be an error of a few centimetres which would be carried forward.

Remark 2.—Usually the slide rules do not contain the values of cot Z. This applies in the case of instruments for which the o of the vertical limb corresponds to the horizontal position of the telescope (Fig. 15); but when the o corresponds

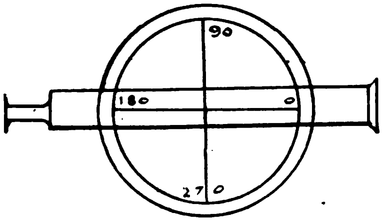


FIG. 15.

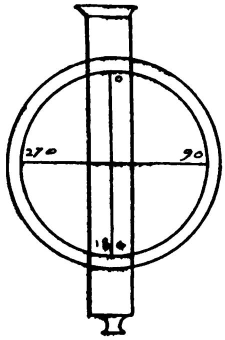


FIG. 16.

to the zenith (Fig. 16) (and this was the case when we established formula 3), we must take the complement of angles between 0° and 90°, and deduct 90° from angles between 90° and 180°.

Remark 3.—The slide rule does not give us the position of the point. For instance, we have:

Corrected distance = 200 m.
$Z = 88° \ 45'$

find *d* cot Z

$$90° - 88° \ 45' = 1° \ 15'.$$

For 11° 15′ and 200 m.,

the slide rule will give us 398

certainly it is not 398 m.
neither is it 39 m. 80
consequently it is 3 m. 98

A little use of the slide rule will accustom us to find out the position of the point.

A table of natural tangents for every 5° is very useful to find the position of the decimal point.

PLOTTING.

When the whole of our tacheometrical book is calculated, we can start plotting. The best plan is to start by plotting the base line roughly on a small scale, so that the top looks north, the left east, the bottom south and the right west. We will then know the width of paper required, and we can arrange our plan symmetrically.

Plotting Base Line.—If plotting the base line with a protractor, we must take from our book a copy of the corrected distances and the angles of the polygonal line.

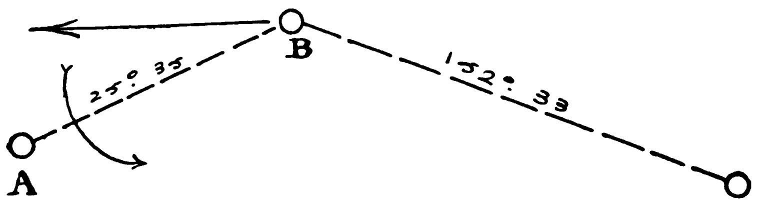


FIG. 17.

The angles between the sides of the polygonal line are measured with the magnectic or true north. We have then to subtract the two bearings to find the included angles.

Example.—

Angle at station B
Bearing B A = 25° 35′
„ B C = 152° 33′
Angle A B C = 152° 33′ − 25° 35′ = 126° 58′

This supposes that the horizontal limb of the telescope is graduated N.W.

When all the distances between the station pegs and the

angles are calculated and written either in a list or on a sketch, we can start the plotting of our base line.

In plotting with the included angles of the base line, we have to use a protractor with arms fitted with a glass disc in the centre.

Plotting by Co-ordinates.—The plotting of the base line with a protractor is very accurate when the station C is near the pricker of the arm or a little outside, but when C is far from the protractor we must draw a straight line up to station peg C.

Suppose the distance B C = 650 in a scale of 1 : 1000, it makes 0 m. 65 centimetres if the semidiameter of the protractor is 30 centimetres. We must draw a straight line of 65 centimetres outside the protractor. A slight difference in the position of the protractor alters the position of peg C. In such cases it is preferable to plot by co-ordinates.

Description of this Method.—Suppose we have to plot a polygonal line A B C D E, etc. All this line may be plotted very accurately when the distances x_1, x_2, x_3, x_4 from the line O X,

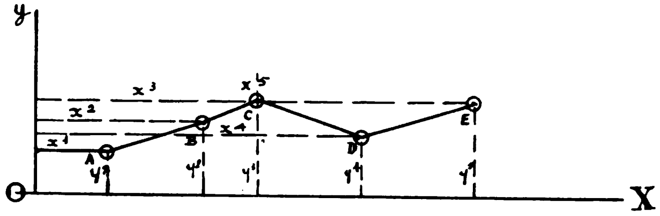


FIG. 18.

and the distances y_1, y_2, y_3, y_4 from the line O Y perpendicular to O X are known, x_1, x_2, x_3, x_4 . . . are called abscissae, y_1, y_2, y_3 y_4 . . . ordinates. Lengths x_1, x_2, x_3, x_4 . . . and y_1, y_2, y_3, y_4 . . . may be calculated when we know the angles of A B, B C, C D with line O X and the distances O A, A B, B C, etc.

We have for the abscissae

$$x_1 = \text{O A} \cos \text{A O X}$$
$$x_2 = x_1 + \text{A B} \cos \text{B A X}$$

$$x_3 = x_2 + \text{B C} \cos \text{C B X}$$
$$x_4 = x_3 + \text{C D} \cos \text{C D X}$$

and for the ordinates

$$y_1 = \text{O A} \sin \text{A O X}$$
$$y_2 = y_1 + \text{A B} \sin \text{B A X}$$
$$y_3 = y_2 + \text{B C} \sin \text{C B X}$$

All these quantities have to be taken as algebraical, viz. positive or negative.

Suppose O X or O Y is the direction of the magnetic north. All these angles A O X, B A X, C B X, etc., are measured, and we have only to correct them according to the variation of the compass.

The following sketch will show the correction to be made, and the way to calculate this correction.

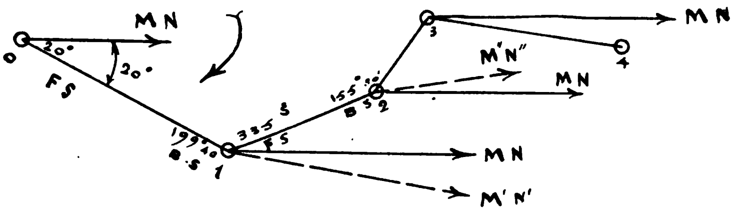


FIG. 19.

The fore-sights of the different sides of base line 0–1–2–3–4 . . . are respectively

$$\begin{aligned} 0\text{–}1 &= 20^\circ \\ 1\text{–}2 &= 335^\circ \\ 2\text{–}3 &= 305^\circ \\ 3\text{–}4 &= 220^\circ \end{aligned}$$

If the compass could be clamped accurately, and there were not magnetic variations, the back-sights of the azimuths would be the same as in column "theoretical."

	Theoretical	Real Reading
1–0 . . .	200° . . .	199° 40′
2–1 . . .	155° . . .	155° 30′
3–2 . . .	125° . . .	125° 50′
4–3 . . .	202° . . .	201° 40′

Thus the azimuth instead of being 200° in the reading 1–0 is 199° 40′; that means that the magnetic north is in the position M′ N′, and the fore-sight of side 1–2 has to be corrected by 20′.

The correct direction of 1–2 is 335° 20′.

At station 2 we ought to have a back-sight reading of 155° 20′, but the reading is 155° 30′. This shows that the magnetic north has again shifted in the direction M″ N″, and the fore-sight reading 2–3 must be altered by 10′. Instead of 305° it is 304° 50′. From the above we can make the following rule.

Rule.—Let us call

F_1 F_2 F_3 F_4 . . . Fn the fore-sight readings of azimuths
B_1 B_2 B_3 B_4 . . . Bn the back-sight readings of azimuths
R_1 R_2 R_3 R_4 . . . Rn the correct azimuths

We have

$$
\begin{aligned}
R_1 &= F_1 \\
R_2 &= F_2 + (R_1 - B_1) \\
R_3 &= F_3 + (R_2 - B_2) \\
R_4 &= F_4 + (R_3 - B_3) \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \\
Rn &= Fn + (R_{n-1} - B_{n-1}).
\end{aligned}
$$

The quantity $(R_{n-1} - B_{n-1})$ is algebraical and is either positive or negative.

In working out the above corrections we suppose two things.

1. That the first azimuth F_1 is correct.

2. That we do not take into consideration the difference of 180° between fore-sight and back-sight readings of azimuths.

Example.—We give an example of this rule in figures.

The first azimuth 20° N E is considered as good. The other azimuths have to be corrected as follows, not taking into consideration the difference of 180°.

Correction of the second azimuth, 335°

$$R_1 - B_1 \text{ or } 20^\circ - 19^\circ\, 40' = 20'$$

Stations	Azimuths		Corrected Azimuths	Distance of Pegs
	Back-Sight	Fore-Sight		
A		20°		
B	199° 40′	335°+	20°	255
C	155° 30′	305°	335° 20′	388
D	125° 50′	22° 10′	304° 50′	428
E	201° 35′	44° 40′	21° 10′	375
F	224° 10′		44° 15′	410

FIG. 20.

consequently the correct azimuth is

$$335^\circ + 20' = 335^\circ\, 20'.$$

Correction of the third azimuth 305°

$$R_2 - B_2 \text{ or } 335^\circ\, 20' - 155^\circ\, 30' = -10'$$

consequently the correct azimuth is

$$305^\circ - 10' = 304^\circ\, 50'.$$

Correction of the fourth azimuth 22° 10′

$$R_3 - B_3 = 304^\circ\, 50' - 125^\circ\, 50' = -1^\circ$$

consequently the correct azimuth is

$$22^\circ\, 10' - 1^\circ = 21^\circ\, 10'.$$

Correction of the fifth azimuth 44° 40′

$$R_4 - B_4 - 21^\circ\, 10' - 21^\circ\, 35' = -25'$$

consequently the correct azimuth is

$$44^\circ 40' - 25' = 44^\circ 15'.$$

We have thus the angles of all the sides of the base line with the magnetic north, or with O X or O Y. Consequently we can calculate the abscissae and ordinates of all the station pegs.

SIGNS OF SINES AND COSINES.

In plotting the abscissae and the ordinates of the station pegs, we have to be very careful about the signs of sines and cosines, as they may be positive or negative. The following precautions must be taken.

The base line must be entirely included in the first quadrant of the circle.

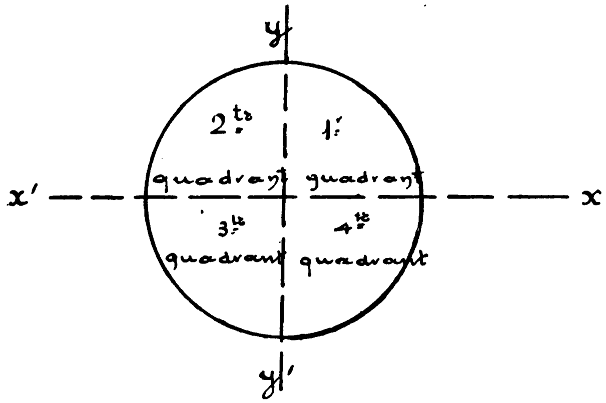


FIG. 21.

This is always possible as we can choose the position of the origin. The small scale plan of station pegs will help us to find this position. All the abscissae on the right of yy' are positive and those on the left are negative.

Similarly, all the ordinates above $x x'$ are positive, and those below $x x'$ negative.

We must always have in mind the following sketch or the following table.

We will show by an example how all these calculations have to be made.

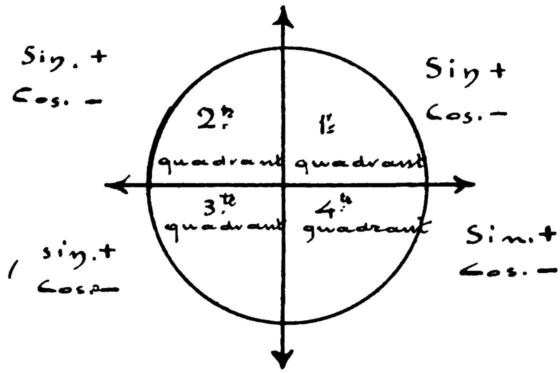


FIG. 22.

	1st Quadrant	2nd Quadrant	3rd Quadrant	4th Quadrant
Sine. . . .	+	+	−	−
Cosine . . .	+	−	−	+

FIG. 23.

We take the polygonal line A B C D E F, of which we calculated the correct angle in Fig. 20.

From the small scale plan we see that the entire line will be included in the first quadrant, if we take for co-ordinates of A, $x = 0$, $y = 500$.

Ox is the direction of the magnetic north.

The graduation of the limb is N E.

See Figs. No. 24 and 24A.

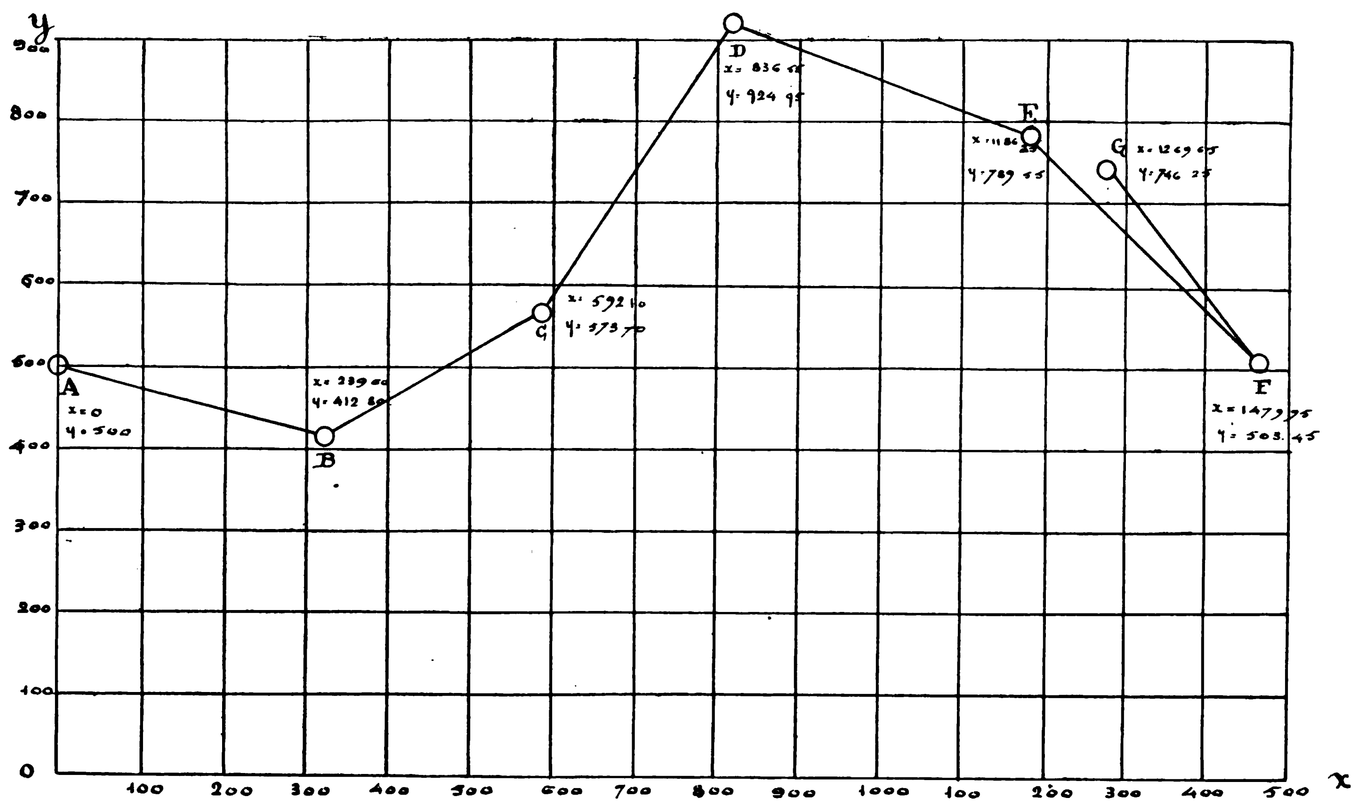

y
x
A
B
C
D
E
G
F
x=0
y=500
y=412.80
x=592.10
y=573.70
x=836.55
y=924.95
y=789.55
x=1269.65
y=746.25
x=1479.75
y=503.45

FIG. 24.

Station Pegs	Azimuth		Corrected Azimuths	Distances	Partials				Total	
					y		x		Y	X
					+	−	+	−		
1	2	3	4	5	6	7	8	9	10	11
A	..	20° 0'		..	..	..	..	..	500·00	000·00
			20° 0'	255	..	87·20	239·60			
B	199° 40'	335° 0'	..	..	..	..	..	..	412·80	239·60
			335° 20'	388	161·90					
C	155° 30'	305° 0'	..	..	..	..	..	..	573·70	522·10
			304° 50'	428	151·25	..	244·45			
D	125° 50'	22° 10'	..	..	..	..	..	..	224·95	836·55
			21° 10'	375	..	135·40	349·70			
E	201° 35'	44° 40'	..	..	..	..	..	..	789·55	1186·25
			44° 15'	410	..	286·10	293·70			
F	223° 52'	..	..	..	..	..	..	..	503·45	1479·95
			130° 47'	322	243·80	..	..	210·30		
G	..	130° 24'	..	..	..	..	..	..	746·25	1269·65

FIG. 24A.

Plotting of the Base Line.—With columns Nos. 10 and 11 the base line may be plotted.

1. Near the bottom of the paper and with a silk thread, stretched tightly between the extreme ends, make punctures

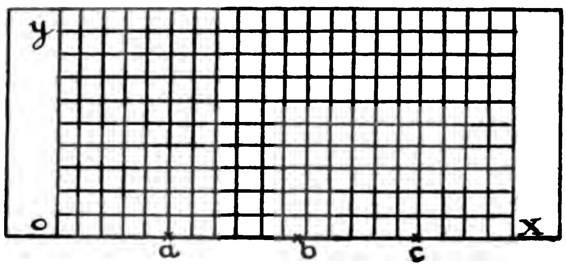


FIG. 25.

a, *b*, *c*, etc., in the paper, and join these with a steel straight edge rule. This line will be O X.

2. Proceed in the same way for O Y, if the line O Y is longer than the straight edge.

3. Take great care that lines OX and OY are perpendicular.

4. At every length of straight edge draw a line perpendicular to O X, and by the known proposition—

$$\text{Hypotenuse} = \sqrt{\text{base}^2 + \text{perp.}^2}$$

you can see that all perpendicular lines are accurately drawn. (Fig. 26.)

5. Divide all the drawing-paper wanted into squares of 10 cm. (or according to the scale of the plan), and plot the

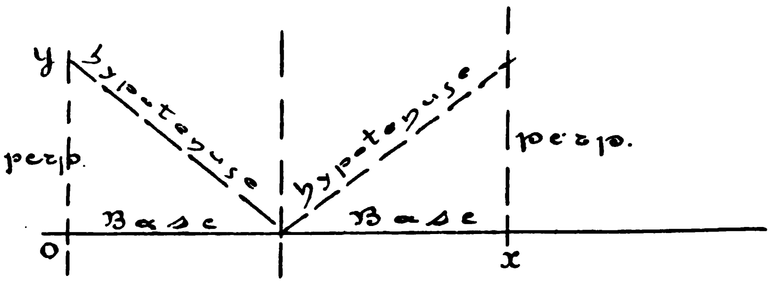


FIG. 26.

positions of the station pegs, using the scale and a small set square perpendicularly to the sides of the square of 10 cm.

6. Measure with the scale each section of the base line and see if they correspond to the lengths written in column No. 5 of co-ordinates. If they do not, there must be a mistake either in the plotting or in the calculation of the co-ordinates.

7. OX and OY and all the parallel lines must be drawn with a very hard, chisel-shaped pencil.

8. All the base line must be drawn with a fine carmine line, and station pegs thus, ⊙ A, before starting the plotting of points.

Note.—In Fig. 23 the partial co-ordinates x and y have been calculated with logarithms, but we can also use the slide rule to make these calculations. We calculated them with a

small slide rule (Tavernier Gravet), and found the following figures.

We put the accurate figure in the second column so as to see the difference.

y or Sines.

By Slide Rule	By Logarithms
87·20	87·20
162·00	161·90
352·00	351·25
135·00	135·40
286·00	286·10
244·00	243·80

FIG. 28.

x or Cosines.

By Slide Rule	By Logarithms
240	239·60
353	352·50
245	244·45
350	349·70
294	298·70
210	210·30

FIG. 29.

The comparison of these figures shows us that the difference is not more than 0 m. 50. For a plan at a scale of 1 : 1000 it is preferable to use logarithms for the accurate calculation of *x* and *y*.

Plotting of Points.—The plotting and inking of the base line completed, we can start plotting the points of the stations.

All the points can be plotted with a protractor.

Special protractors made in cardboard are the most convenient for this purpose.

In Fig. 30 we give a design of one of these protractors

They must be graduated in the same direction as the limb of the instrument.

On each side of the centre, a scale in accordance with the scale of the plan wanted is drawn. The centre itself is a little hole through which we can prick a fine needle and fix the protractor in the station peg.

The book-keeper with his tacheometrical book helps the surveyor or the leader who is in charge of the plan, and dictates to him.

1. The azimuths or horizontal angles.

2. The distances of points from station pegs.
3. The reduced levels which are written in pencil.

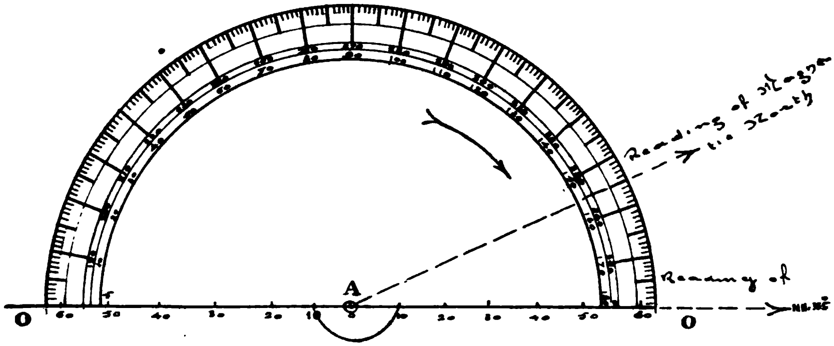


FIG. 30.

Example.—Plot point 1 of station A, azimuth 335°.

We put the angle 335° in the direction of the observed magnetic north. The point is then in the direction of the line O - O.

From the centre of the protractor to the right we mark 58 mm., if the scale is 1 : 1000, and thus we have only the level to write.

With a little practice we cannot make any mistakes; plotting, for instance, point 1 in the direction of 0° + 180°, we must always look in which quadrant of the circle the angles are included. In this way we can plot all the points from station A.

When station A is finished, we remove the protractor to station B, and fixing it with a needle, we start to draw the direction of the magnetic north, and continue to plot the points. When all the plan is finished, we ink the positions of the points and the reduced levels. The plan is then ready for contouring.

CONTOURING.

Contours are usually drawn every 1, 2, 5 or 10 metres, according to the object of the plan and its scale.

Problem.—Given the reduced levels between two points, draw the contours between them.

Suppose we have four points *a, b, c, d* with the following levels

$$a = 14 \cdot 32$$
$$b = 11 \cdot 40$$
$$c = 7 \cdot 80$$
$$d = 9 \cdot 30$$

Their respective positions are shown in Fig. 31.

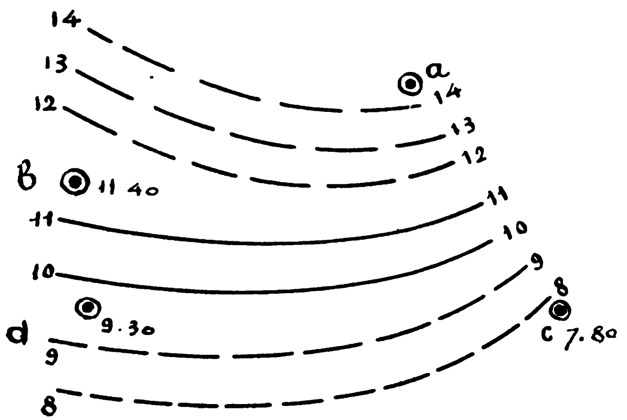


FIG. 31.

We want to draw our contours every metre.

On a small paper we plot the vertical interval of contours according to the scale of the plan.

The line A B is drawn at the height 7·80 and equal to *a c*.

At point B we draw a perpendicular line to A B up to 14·32.

We join 14·32 to A and project points 9, 10, 11, 12, 13, 14 to line A B. This gives us the horizontal equivalents of contours between *a* and *c*.

A quicker method is to have the lines 7, 8, 9 plotted on a tracing cloth. Fix 7·80 with a needle to point C, then moving the tracing cloth, place *d* between 14 and 15 at 14·32. Join

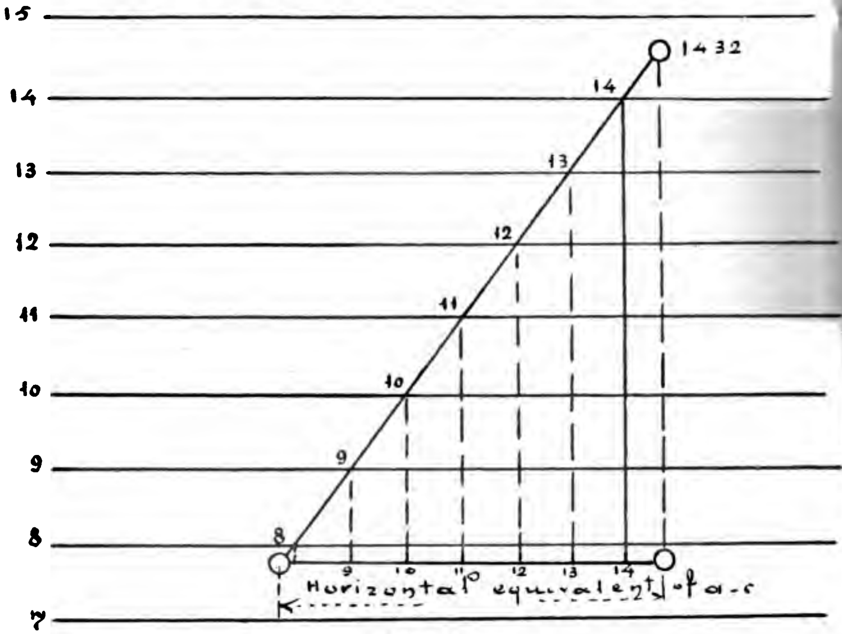


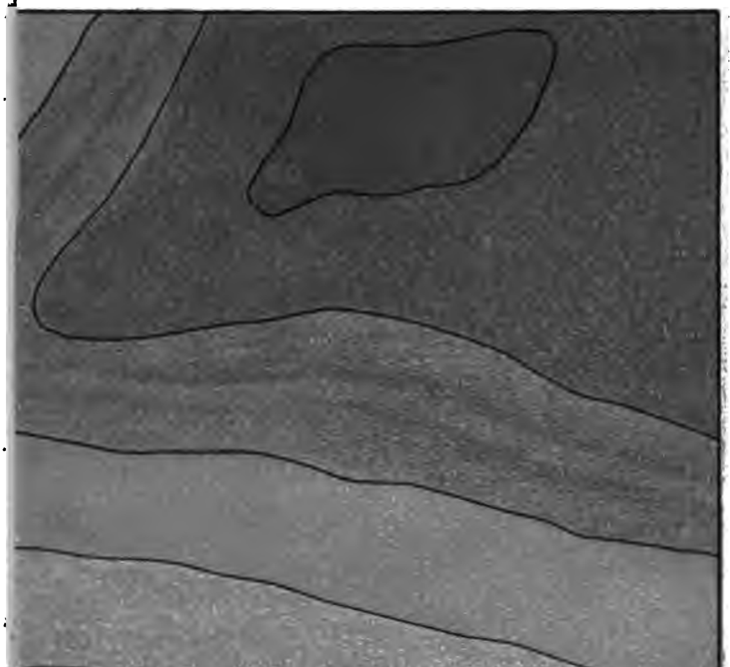
FIG. 32.

with a line *d* and *c*, and with a pricker puncture the points where line *d c* cuts across the lines 8, 9, 10, 11, 12, 13, 14. Those points belong to the contours 8, 9, 10, 11, 12, 13, 14. This method supposes that the declivities of the ground are regular between *d* and *c*.

The surveyor and the leader must be very careful that this condition is fulfilled in field work.

When all the contours are finished, we ink them with burnt sienna.

Note.—In small scale plans we can show very neatly the relief of the ground by colouring the contours, as shown in Fig. 33. The lower contours are lighter, and as we ascend they become gradually darker.



120
110
100
90

Recapitulation of Tacheometrical Surveying.

Field Work.—When adjusting the instrument it is important that all friction should be avoided as much as possible.

Check on the spot azimuths, distances, and levels between pegs.

Azimuthal angles must not differ more than 20′.

Distances not more than 0 m. 75.

Levels not more than 0 m. 06.

In surveying points the leader must be very careful about the position of the staffs, and must keep his field book neatly.

Office Work.—1. With the assistance of the book-keeper calculate columns Nos. 10 and 11. Then the book-keeper himself can finish the calculations of columns Nos. 8, 9, 10, 11, 12, 13, 14, 15.

2. Plot the base line roughly in a small scale to see the paper required, and find position of O X and O Y.

3. Calculate the co-ordinates.

4. Plot the base line.

5. Plot the points.

6. Ink the points.

7. Contour.

Similar Instruments to the Tacheometer.—As we have already said, the tacheometer is a usual theodolite fitted with stadia. Another similar instrument is the Fennel tacheometer. In this model the vertical limb is suppressed, and instead of reading the zenithal angle Z, we can, by a sort of slide rule fitted in the place of the vertical limb, read the correct distance at once and d cot Z.

Another improvement is to mark the staffs at the height of the instrument. The mean wire must strike this mark.

Referring to formula 3

$$H = h_1 + d \cot Z - h$$

If in this formula we make $h_1 = h$
we have

$$H = d \cot Z$$

and the difference of level is calculated at once, and the work of the reduction of the book is considerably lightened.

Practically we can never have $h_1 = h$, as the height of the instrument above the ground varies according to the spread given to the legs, and it is difficult to set it and have the height of the telescope equal to the height of the mark on the staff.

Usually the mark is 1 m. 50 above the foot of the staff, and the height of the tacheometer is between 1 m. 35 and 1 m. 55; thus we have to strike the staff 15 centimetres below or 5 centimetres above the mark.

In other instruments the horizontal limb is also suppressed and replaced by an ordinary plane table. The points are thus plotted on the ground with their levels, and the rest of the work, viz. contouring, is done in the office.

All these instruments reduce or even nearly suppress the office work, but they increase the field work. For example, in the case of a Fennel tacheometer we reckon that we can only survey $\frac{2}{3}$ of the points we could otherwise survey with an ordinary tacheometer.

Field work is much more difficult than office work and depends on the weather, while office work can be done during the rainy or windy days when surveying is impossible.

For these reasons we should give preference to the ordinary tacheometer, by which field work is as quickly accomplished as possible.

Many other instruments derive from the tacheometer.

The problem of measuring distances without chaining is very attractive in itself, and a great many surveyors have tried to solve it. You can find in many advertisements different instruments which pretend to measure distances with the same accuracy as good chaining. As long as they are based on stadia we cannot rely on them.

CHAPTER V.

TACHEOMETRICAL SURVEYING.

IN Chapter IV. on field work we described the method of traversing or surveying along a polygonal line, whilst here we will try to explain the method of tacheometer surveying, depending upon a system of triangles.

Tacheometrical traversing is the surveying of a zone of 700 m. wide (350 m. each side of the base line), while tacheometrical surveying is applicable to a zone of 10 or 15 miles wide.

A triangulation is necessary to determine various points.

This triangulation must be made, taking the following precautions.

1. The sides of the triangles must be of an average length of 700–1000 m. The summits of triangles must be, if possible, in the features of the ground, such as boundaries, fences, roads, etc.

2. The angles of the triangles must be neither too acute nor too obtuse; the minimum for acute is 25°, and for obtuse 125°, but a preference should always be given to equilateral triangles.

3. Every angle has to be measured with an approximation of 10″.

4. All the three angles of a triangle must be measured and their sum must not differ more than 10″ from 180°.

5. At the two ends of the survey two base lines of about 500 m. in length must be very carefully measured with an approximation of 2½ centimetres.

6. In countries where reliable trigonometrical surveys exist, the other triangles may be joined to them in order to have a satisfactory check.

Such surveys exist in all Europe, the English colonies, some of the French colonies, North America, some countries of South America, Japan, etc.

Particulars concerning this work will be found in special books dealing with this subject, especially concerning the method of measuring angles, base line, etc.

When the field work is completed, our triangulation has to be plotted out roughly and all the angles slightly corrected by a few seconds in order to have the three angles of a triangle equal to 180° and angles round A equal to 360°.

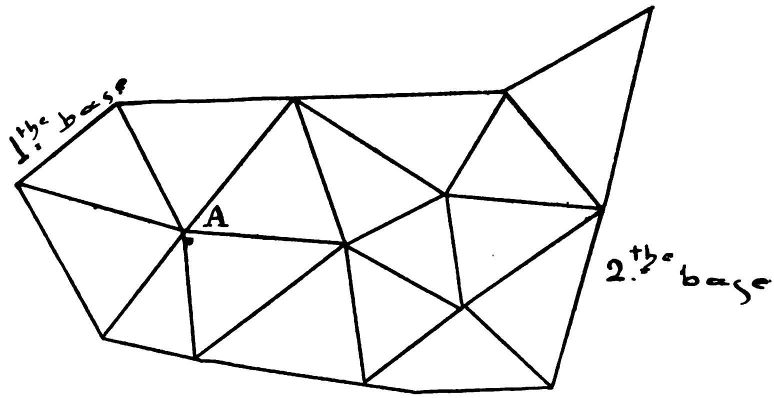


FIG. 34.

The calculation of all the sides may be commenced, starting from first base A and going towards second base.

The difference between the measured and the calculated length of base must not exceed 2½ cm., or an inch.

In order to plot all the triangulation with co-ordinates referring to the north, the azimuth of one side of a triangle must be determined. The azimuth of another side must also be determined as a possible check.

With such a triangulation we have five or six very accurate points in every square mile, and we can proceed to the surveying of all the features wanted, such as boundaries, fences, roads, streams, canals, etc.

We consider the following the best way to proceed.

Plant the tacheometer in an important point of the ground, and find its position by the readings of the two angles to the three summits of the triangulation.

We have thus to solve the following problem.

Problem.—Knowing angle A B C and the length of sides A B and B C, find the position of T from the readings of angles

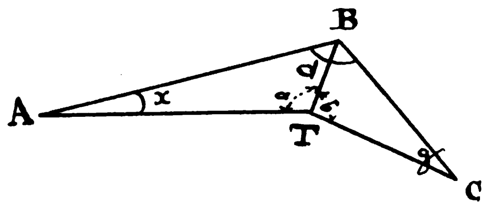


FIG. 35.

a and *b*. Angles A B C and length of sides A B and B C are given from the triangulation.

With a three arms protractor, as Fig. 36, we make angles *a* and *b*, and shift it in order that the arms of the protractor

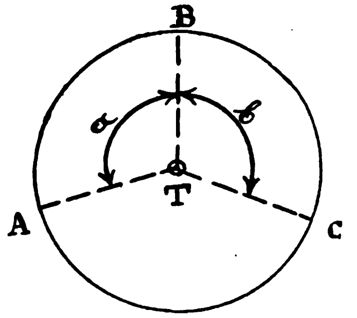


FIG. 36.

strike points A B C, but this method is not very accurate; it is preferable to find the position of T by calculation.

In Fig. 35 we have

$$x + y + a + b + d = 360^\circ$$

consequently

(1) $$x + y = 360^\circ - (a + b + d)$$

or

$$\frac{x + y}{2} = 180^\circ - \frac{(a + b + d)}{2}$$

We have also

$$\frac{\mathrm{BT}}{\sin x} = \frac{\mathrm{AB}}{\sin a}$$

and

$$\frac{\mathrm{BT}}{\sin y} = \frac{\mathrm{BC}}{\sin b}$$

dividing the second equation by the first we find

(2) $$\frac{\sin x}{\sin y} = \frac{\mathrm{BC} \sin a}{\mathrm{AB} \sin b}$$

Thus between x and y we have two equations, and can find x and y, making

$$\tan m = \frac{\sin x}{\sin y} = \frac{\mathrm{BC} \sin a}{\mathrm{AB} \sin b}$$

$$\frac{1 + \tan m}{1 - \tan m} = \frac{\sin x + \sin y}{\sin x - \sin y} = \frac{\tan \frac{1}{2}(x + y)}{\tan \frac{1}{2}(x - y)}$$

$$\tan \tfrac{1}{2}(x - y) = \frac{\tan \frac{1}{2}(x + y)(1 - \tan m)}{1 + \tan m}$$

but

$$\frac{1 - \tan m}{1 + \tan m} = \tan(45^\circ - m)$$

consequently

$$\tan \tfrac{1}{2}(x - y) = \tan \tfrac{1}{2}(x + y) \tan(45^\circ - m).$$

Knowing $x - y$ and $x + y$ we calculate x and y, and thus we can calculate an ordinary oblique-angled triangle.

As $\tan 45^\circ = 1$, $\tan m$ must always be less than 1.

In the case $m = 45^\circ$

$$\tan(45^\circ - m) = 0.$$

the problem is impossible, and point T is situated in the circle passing through the three points A, B, C, therefore we must always try and be as far as possible from the circle A B C. Point T must be in the interior of triangle A B C.

Example.—

$$
\begin{aligned}
\text{A B} &= 502 \text{ m. } 40. \\
\text{B C} &= 690 \text{ m. } 80. \\
\text{A B C} &= 82^\circ\ 35'\ 40'' \\
a &= 88^\circ\ 31 \\
b &= 116^\circ\ 48.
\end{aligned}
$$

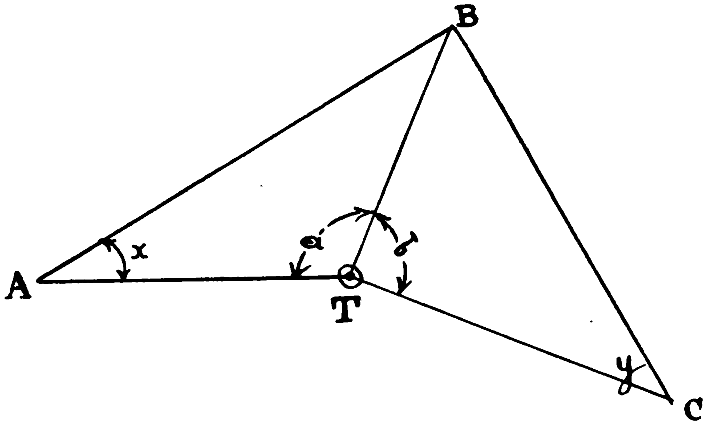


FIG. 37.

A B, B C, and angle A B C are taken from the triangulation, the angles a and b are measured from the ground.

Plot the position of point T.

$$\tan m = \frac{\sin y}{\sin x} = \frac{\text{A B} \sin b}{\text{B C} \sin a}$$

$$
\begin{array}{rlrl}
\log \text{A B} = & 2{\cdot}70105 & \log \text{B C} = & 2{\cdot}83935 \\
\log \sin b = & \bar{1}{\cdot}95065 & \log \sin a = & \bar{1}{\cdot}99985 \\
 & 2{\cdot}65170 & & 2{\cdot}83920 \\
= & \bar{3}{\cdot}16080 & & \bar{3}{\cdot}16080 \\
\log \tan m = & \bar{1}{\cdot}81250 & &
\end{array}
$$

$$m = 33^\circ$$

$$(45^\circ - m) = 12^\circ$$

$$\log \tan 12^\circ = \bar{1}\cdot 32747$$

$$\frac{x+y}{2} = 180^\circ - \frac{(82^\circ\ 35'\ 40'' + 88^\circ\ 31' + 116^\circ\ 48')}{2}$$

$$\frac{x+y}{2} = 36^\circ\ 2'\ 40'' \quad \log \tan = \bar{1}\cdot 86197$$

$$\log \tan.\ 12^\circ = \underline{\bar{1}\cdot 32747}$$

$$\log \tan \tfrac{1}{2}(x-y) = \bar{1}\cdot 18944$$

$$\frac{x-y}{2} = 8^\circ\ 47'\ 40''$$

$$\frac{x+y}{2} = 36^\circ\ 2'\ 40'' \qquad 36^\circ\ 2'\ 40''$$

$$\frac{x-y}{2} = \underline{8^\circ\ 47'\ 40''} \qquad \underline{8^\circ\ 47'\ 40''}$$

$$x = 44^\circ\ 50'\ 20'' \qquad y = 27^\circ\ 15'\ 0''$$

Knowing x and y we can calculate triangles A B C or B C T—of course the length of common side B T must be the same.

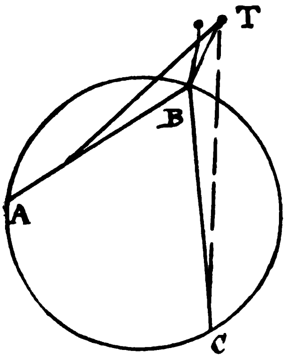


FIG. 38.

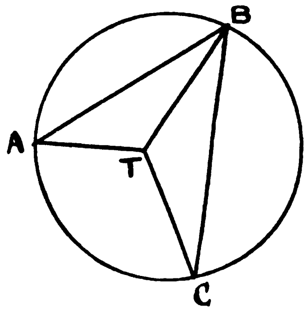


FIG. 39.

All this calculation must be done very carefully, as there is no check.

If we want to have a check, we must measure the angle with another summit of triangulation.

Point T or the instrument peg must always be as far as possible from the circle passing through points, as shown in Figs. 38 and 39.

If point T is on the circle A B C, as in Fig. 40, all the points of the circle A B C are positions of point T, and the solution is impossible.

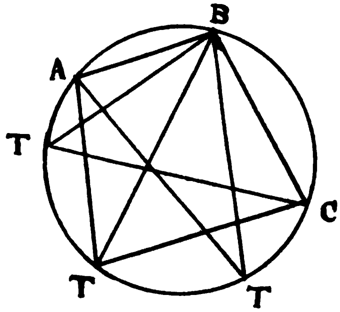


FIG. 40.

This method is not usually employed by surveyors, the calculation being longer than by other methods, but it has the advantage of enabling us to choose the position of all station pegs which are independent of one another, and an error of one of them has no effect on the others.

Besides that, no chaining is required. For these reasons we strongly recommend its employ.

A surveyor working methodically with his assistant can calculate the position of 15 points as T by the above method in two hours.

Assume that we need a station T in a square of 300 m. × 300 m., twenty-five stations every square mile will be wanted.

All this can be carried out with five decimal logarithms, and for distances between 600 and 800 m. the angles need not be measured with an approximation of more than one minute.

The angles a and b may be measured with a sextant, or reflection circle fitted with a special tripod for this purpose.

GRAPHICAL METHOD TO FIND POINT T.

Suppose
$$ABC = 140^\circ$$
$$AB = 1000 \text{ m.}$$
$$BC = 800 \text{ m.}$$
$$a = 65^\circ$$
$$b = 58^\circ$$
find position of point T.

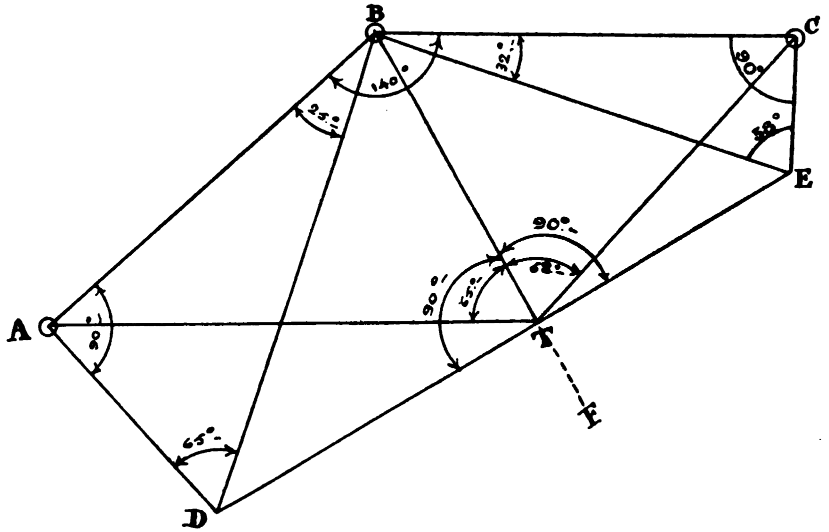


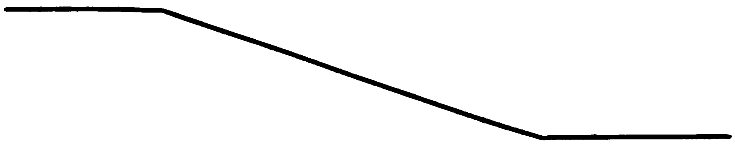
FIG. 41.

Draw A D perpendicular to A B
„ C E „ to B C
Angle A B D = 90° − 65° = 25°
„ C B E = 90° − 58° = 32°
We find thus points D and E
Join D and E, draw perpendicular B F to D E
Intersection of lines B F and D E is point T.

Having thus fixed the position of point T, we can start surveying all the necessary topographical details, such as fences, boundaries, canals, roads, etc. All this survey may be

performed with the radiating system, using the tacheometer and according to the following rules.

1. The scale of the stadia must be 1 : 50 ; or the measuring angle of 1° 8′ 45″.



2. The maximum distance to be taken must not exceed 150 metres (the accuracy of length is very nearly 0 m. 20).

3. The zenithal angles must be between 88° and 92°, as steeper readings are not accurate.

4. In order to have the best results with stadia, it is important to strike all the features of the ground with an acute radiating angle. Fig. 42 will explain the meaning of this.

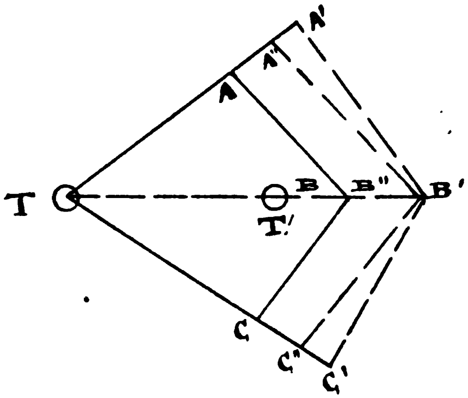


FIG. 42.

Suppose we have to survey from station peg T boundary line A B C.

Should our measurement with the stadia be 0 m. 50 wrong, instead of having A B C we have a line like A′ B′ C′.

If our station peg was at T′ and our stadia wrong of 0 m. 50, the new line A″ B″ C″ would be nearer to the boundary line A B C. The accuracy is in inverse proportion to the sines of the angle of boundary and of radiating lines T A, T B, T C.

Thus supposing the error of the stadia is 0 m. 50, the total error for different angles is:

0 m. 50 × sin 10°	= 0 m. 50 × 0·77	= 0 m. 08
0 m. 50 × sin 20°	= 0 m. 50 × 0·34	= 0 m. 17
0 m. 50 × sin 30°	= 0 m. 50 × 0·50	= 0 m. 25
0 m. 50 × sin 45°	= 0 m. 50 × 0·70	= 0 m. 35
0 m. 50 × sin 60°	= 0 m. 50 × 0·87	= 0 m. 43
0 m. 50 × sin 90°	= 0 m. 50 × 1·00	= 0 m. 50

We consider that when this work is properly carried out, the trigonometrical points are reckoned with an approximation of 3–4 centimetres; the subsidiary stations, such as T, with 8–10 centimetres, and the detail points with an approximation of 0 m. 20 to 0 m. 25.

Except in the case of a town survey this approximation is sufficient, even for a very valuable ground.

Recapitulation.

All this method can be recapitulated thus:—

1. Establishment of trigonometrical summits of tertiary triangulation (sides between 700 m. and 1000 m.).

All the ordinary methods employed in tertiary triangulation hold good (see special books dealing with this subject).

2. Reconnoitring of the ground, and measuring the two angles *a* and *b*. Stations such as T may be considered as the summits of a quaternary triangulation.

3. From stations of tertiary and quaternary triangulation, take all the details required of the map by means of radiated distances, and all the levels if the latter is necessary.

Comparison with Other Methods.—We consider this method superior to other methods, such as traversing from one summit to another, for the following reasons.

1. All stations such as T are independent one from the other, and an error in one station does not affect the others.

2. The work is more methodically carried out.

3. The field work is considerably reduced.

4. We avoid cutting trees, fences, etc., and less damage is done to the crops.

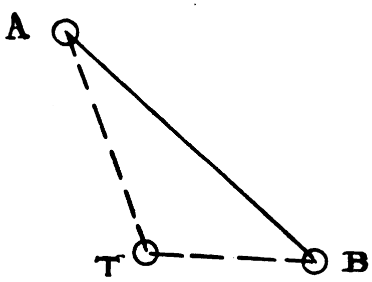


FIG. 49.

Only the office work is slightly increased, owing to the calculations of stations of quaternary triangulation, but the accuracy is very nearly that of a good chaining.

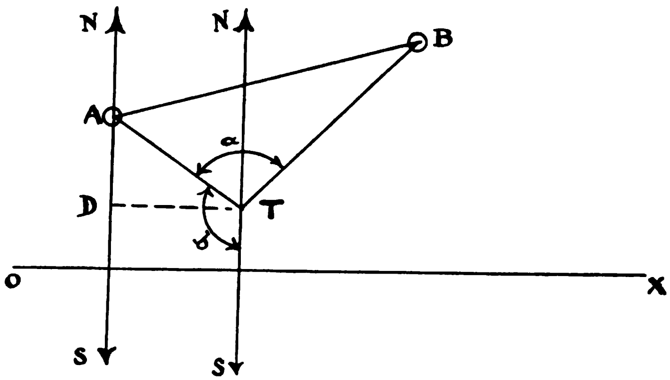


FIG. 50.

Note 1.—In a great many cases the calculation may be reduced, for instance, should stations such as T be near the summit of the triangulation, we can chain the distance T B

and measure the angle A T B. We calculate triangle A T B, knowing two sides A B and T B and one angle A T B. The calculation in this manner is reduced. (Fig. 49.)

Note 2.—Instead of measuring angles a and b with two sides of the triangulation, we can measure angle a with two summits A and B, and angle b with true north or south. (Fig. 50.)

In another chapter we will explain the method of finding angle b or the azimuth of side T A.

As the triangulation summits are plotted by co-ordinates, we know the azimuth of A B. Let us call c this azimuth or angle S A B.

$$\mathrm{SAT} = 180^\circ - b$$
$$\mathrm{SAB} = c$$

consequently

$$\mathrm{SAB} - \mathrm{SAT} = \mathrm{TAB} = c - (180 - b) = c + b - 180^\circ$$

When angle T A B is known, we can calculate A D and D T, and plot station T by its co-ordinates x and y.

APPENDIX I.

DETERMINING OF THE AZIMUTH, ETC.

THE tacheometrical surveyor has often to determine the azimuth of a side of the base line either in order to have a reliable orientation for his plan or else to enable him to check the horizontal angles. When working in rough countries where there is no ordnance map, the surveyor has also to determine the hour-angle and the latitude.

Azimuth.—The azimuth of a side is the angle between the meridian and the side.

The azimuth may be reckoned from N. or S., N.W. or N.E., S.W. or S.E., and may be determined either by observation of the sun or of the stars.

BY OBSERVATION OF THE ZENITHAL ANGLE OF THE SUN.

Suppose the position of the surveyor is at point O. Z, zenith ; P, pole ; S, sun (Fig. 51) ; in the spherical triangle P S Z pole, zenith, sun.

Side P Z is the complement of latitude which must be known or determined by a previous observation.

Side P S is the polar distance.

The nautical almanack gives the apparent declination of the sun's centre at mean noon Greenwich, for every day of the year. To find the polar distance, we add 90° if the declination is south, and we subtract from 90° if the declination is north. S Z is the measured zenithal angle.

Angle A is the azimuth.

Angle H is the hour-angle.

We now know the three sides of triangle P S Z.

Making $PZ = l$ (complement of latitude).
$SZ = z$
$PS = d$

and $l + z + d = 2p$

we can find by the equation

$$\cos^2 \tfrac{1}{2} A = \sin p \sin (p - d) \operatorname{cosec} z \operatorname{cosec} l.$$

Then in order to calculate A we must know

1. The exact latitude.
2. The approximate longitude.

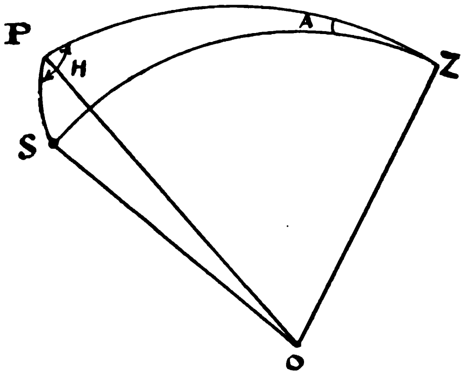


FIG. 51.

The column "Variation in one hour" facilitates the reduction of the quantities from apparent noon to any other time.

The zenithal angle has to be corrected by

1. The sun's parallax and
2. The atmospherical refraction.

The nautical almanack gives the sun's semi-diameter for every day; this we must take into consideration when we observe one of the edges of the sun.

Example.—We observe the zenithal angle of the sun which, when corrected for the parallax, refraction, and semi-diameter, was found equal to 60° 13'.

The latitude is 31° 29'.

The polar distance of the sun is 106° 54′. We find in the nautical almanack that the declination is south and equal to 16° 54′ for the date, south.

Complement of latitude	$l =$	59° 31′	log cosec = 0·06461
Zenithal angle	$Z =$	60° 13′	log cosec = 0·06153
Polar distance	$d =$	106° 54′	
	$2p =$	226° 38′	
	$p =$	113° 19′	log sin = $\bar{1}$·96300
	$p - d =$	6° 25′	log sin = $\bar{1}$·04828
			log cos² ½ A = $\bar{1}$·13742

$$\log \cos \tfrac{1}{2} A = \bar{1}\cdot 56871$$
$$\tfrac{1}{2} A = 68^\circ\ 15'\ 30''$$
$$A = 136^\circ\ 31'$$

This angle A is the included angle between the sun's centre and the north.

By Observation of the Stars.

The same method may be employed by observation of the stars. In pages "Apparent places of the stars" the nautical almanack contains the declinations of 460 stars, any of which may be employed, and this calculation is easier than calculation with the sun, as there is no correction of parallax and semi-diameter.

By Observation of the Polaris.

The French nautical almanack (Connaissance des temps) contains a table giving the azimuths of the polars at any time and for any latitude between 0° and 65° north, when the hour-angle is known.

The hour-angle is equal to the sidereal time minus the right ascension.

The nautical almanack gives the sidereal time at mean noon Greenwich for every day of the year, from which it is

easy to find the local sidereal time. In pages "Apparent places of stars" the right ascension is given; consequently with the latitude and the hour-angle we can find the azimuth of the polaris at any required time.

The latitude should be known with an approximation of 10 minutes and the hour-angle with an approximation of 2 or 3 minutes.

The azimuth of the polaris or of any circumpolar star may be calculated, when the hour-angle is known, by the following formula

$$(a) \qquad \cot A z = \frac{\cot H \sin (l - x)}{\sin x}$$

H the hour-angle

l the complement of latitude.

x an auxiliary angle determined by the equation

$$\tan x = \cos H \tan d$$

d is the polar distance of the polaris given in the almanack by subtracting the declination from 90°.

DETERMINING THE HOUR-ANGLE.

1. By observation of the zenithal angle in the triangle pole-zenith-star.

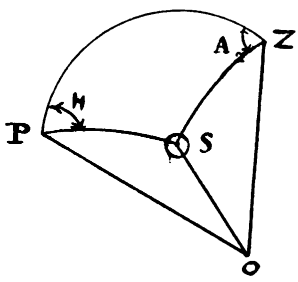


FIG 52.

Knowing the three sides we can determine angle H by formula

$$\tan z \tfrac{1}{2} H = \sin (p - l) \sin (p - d) \operatorname{cosec} p \operatorname{cosec} (p - z).$$

$$l = Pz \qquad 2p = l + d + z$$
$$d = PS \qquad p = \frac{l + d + z}{2}$$
$$Z = Sz$$

If S is the sun, angle H is the apparent time, which we can convert into mean time with the equation of time.

2. By observation of the azimuth.

The hour-angle of the sun or of a star may be determined, when the azimuth is known by formula

$$\sin(H + x) = \sin x \cot d \tan l.$$

x is an auxiliary angle given by equation

$$\tan x = \cos l \tan A\,z.$$

Determining of the Latitude.

The latitude may be determined—

1. By the observation of the pole star out of the meridian, as described in the nautical almanack.

2. By the observation of the zenithal angle of any star at local upper transit.

The nautical almanack in chapter "Apparent places of stars" contains the right ascensions and the declinations of 460 stars, and any of them may be employed.

Of course the zenithal angle has to be corrected for the refraction.

APPENDIX II.

ELEMENTARY THEORY OF STADIA.

O optic centre of the object-glass. (Fig. 53.)
N the position of the staff.
C and D the extreme wires of the diaphragm.
C and D are in the focus of the object-glass and the eye-piece.

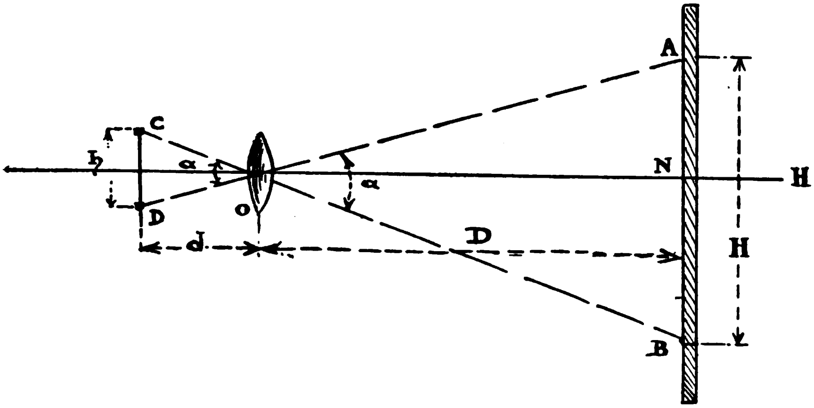


FIG. 53.

In the triangles O A B and O C D we have

$$\frac{D}{d} = \frac{H}{h}$$

consequently

$$D = d\frac{H}{h}$$

H is the difference of readings of the extreme wires.

$\frac{d}{h}$ is the scale of the stadia,

but

$$h = 2\,d \tan \frac{a}{2}.$$

As angle a is very small

$$h = d \tan a$$

$$\begin{aligned} a &= 1^\circ\, 8'\, 45'' \text{ for a scale of } 1:50 \\ a &= 34'\, 22'' \text{ for a scale of } 1:100 \\ a &= 17'\, 11'' \text{ for a scale of } 1:200 \end{aligned}$$

Then the scale of the stadia is

$$\frac{h}{d} = \frac{d}{d \tan a} = \frac{1}{\tan a} = \cot a$$

a is the measuring angle of the stadia.

From Fig. 53 we see that the distances of the staff are reckoned from the centre of the object-glass, and not from the centre of the instrument.

We have then to add to the calculated distance, the distance between the centre of the object glass and the centre of the instrument. This distance in the usual instruments is equal to 0 m. 15 to 0 m. 20—say 0 m. 20.

Between the quantities d and D we have the following equation according to elementary optics.

$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f}$$

f is the focal distance of the object glass, or in the ratio $\frac{d}{h}$ scale of the stadia, h is the distance of the wires and a constant quantity.

But d depends on D

$$\frac{D}{d} = \frac{H}{h} = \frac{D-f}{f}$$

for another distance D_1 we have a reading H_1. Thus

$$\frac{H_1}{h} = \frac{D_1-f}{f}$$

therefore

$$\frac{H}{H_1} = \frac{D-f}{D_1-f}$$

This equation shows that the distances D, D_1, are not reckoned from the centre of the object glass, but from a point situated at a distance f from the object glass.

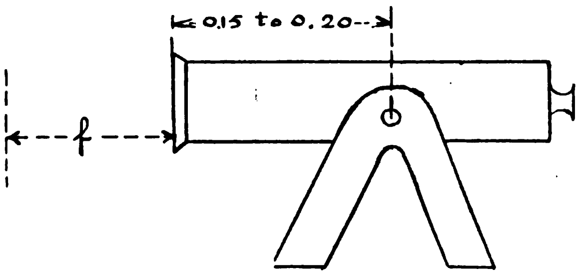


FIG. 54.

f is equal to 0 m. 25 to 0 m. 32 in the usual theodolites and tacheometers, say 0 m. 30.

Every distance has to be corrected of

$$0 \text{ m. } 20 + 0 \text{ m. } 30 = 0 \text{ m. } 50$$

Instead of correcting all the distances of nearly 0 m. 50, Mr. Porro introduced a considerable improvement by fitting the telescope with another lens, avoiding thus the correction of 0 m. 50.

This lens is fixed between the object glass and its focus.

O object glass. (Fig. 55.)

F position of the focus of the object glass.

O′ Porro's lens.

F′ position of its focus.

f focal distance of lens O.

f' focal distance of lens O′.

$m\ n$ the stadia wires.

x the distance of the two lenses.

S S′ the position of the staff.

Point m of the superior wire strikes lens O′ at m'.

There it is refracted and emerges to go through focus F′, strikes lens O at m'', there it is again refracted and strikes the staff at m'''.

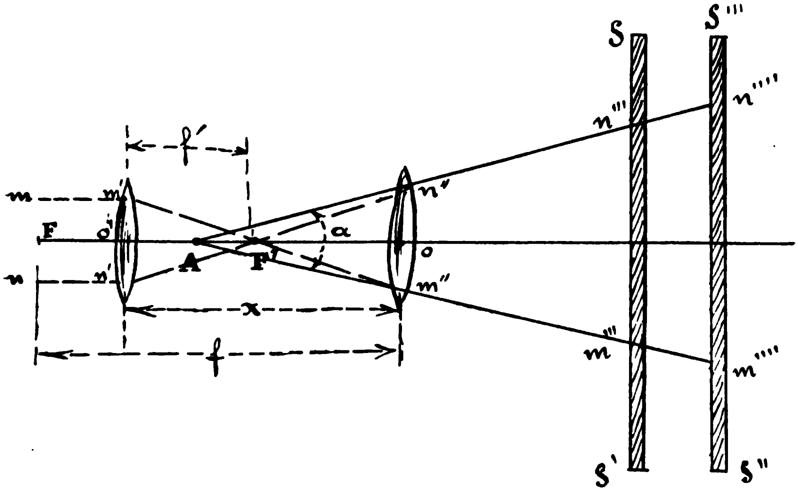


Fig. 55.

Point n of the inferior wire *idem* strikes the staff at n'''.

The two lines $m''\,m'''$ and $n''\,n'''$ join the line O O′ at a certain point A, from which the distances are reckoned.

Should the position of the staff be S″ S‴, the two lines $m''\,m''''$ and $n''\,n''''$ will always join at point A.

The angle of lines A m''' and A n''' is the measuring angle.

The position of point A depends on x and f'.

Judiciously choosing these quantities, viz. the position of lens O′ and its shape, we can manage to have point A in the apex of the tacheometer.

The instruments fitted with Porro's lens have a special screw and key to adjust the lens. In the other instruments we have to take into consideration the correction of 0 m. 20 + 0 m. 30. 0 m. 20 is the distance of the object glass to the centre of the instrument, and 0 m. 30 the focal distance of the object glass.

APPENDIX III.

TABLES FOR THE REDUCTION OF DISTANCES.

COMPUTED BY C. XYDIS.

These tables are useful for finding the corrected distance on sloping ground.

The vertical argument is the difference of readings of the extreme wires.

The horizontal argument is the zenithal angle.

If d is the difference of readings and z the zenithal angle the corrected distance is $d \sin^2 z$.

The tables contain the quantity to be subtracted from the readings in order to have the corrected distance.

Example.—

Difference of readings . . .	255
Zenithal angle	73° 50′

the quantity to be subtracted is 19·77 as shown in Table III. and the corrected distance is

$$255 - 19{\cdot}77 = 235{\cdot}23$$

Those tables are computed for distances of every 2 metres or yards from 30 to 120, and of every 5 metres or yards from 120 to 300. Should the distance or zenithal angle not be included in the tables a slight proportional correction is to be made.

The error cannot be more than 0 m. 10 which is quite unimportant in tacheometrical traversing.

These tables are much more accurate than an ordinary slide rule or diagram.

I.—Tables for the

	87° 40′	87° 20′	87° 0′	86° 40′	86° 20′	86° 0′	85° 40′	85° 20′	85° 0′	84° 40′	84° 20′	84° 0′
	2° 20′	2° 40′	3° 0′	3° 20′	3° 40′	4° 0′	4° 20′	4° 40′	5° 0′	5° 20′	5° 40′	6° 0′
30	0·05	0·06	0·08	0·11	0·13	0·15	0·17	0·20	0·23	0·26	0·29	0·33
2	0·05	0·06	0·08	0·11	0·13	0·15	0·18	0·21	0·24	0·27	0·31	0·35
4	0·05	0·07	0·09	0·12	0·14	0·16	0·19	0·22	0·25	0·29	0·33	0·37
6	0·05	0·07	0·10	0·12	0·14	0·17	0·20	0·23	0·26	0·30	0·35	0·39
8	0·06	0·08	0·10	0·12	0·15	0·18	0·21	0·24	0·28	0·32	0·37	0·41
40	0·06	0·08	0·11	0·13	0·16	0·19	0·22	0·26	0·30	0·34	0·39	0·43
2	0·06	0·09	0·11	0·14	0·17	0·20	0·25	0·30	0·33	0·36	0·41	0·45
4	0·07	0·10	0·12	0·15	0·18	0·21	0·25	0·30	0·35	0·38	0·43	0·47
6	0·07	0·10	0·12	0·15	0·18	0·22	0·26	0·31	0·36	0·40	0·45	0·49
8	0·08	0·11	0·13	0·16	0·19	0·23	0·27	0·32	0·37	0·42	0·47	0·51
50	0·08	0·11	0·13	0·16	0·20	0·24	0·28	0·33	0·38	0·44	0·49	0·54
2	0·08	0·11	0·13	0·17	0·21	0·25	0·29	0·33	0·39	0·45	0·50	0·56
4	0·09	0·12	0·14	0·18	0·22	0·26	0·30	0·34	0·40	0·47	0·52	0·58
6	0·09	0·12	0·14	0·18	0·22	0·27	0·32	0·35	0·41	0·49	0·54	0·60
8	0·09	0·13	0·15	0·19	0·23	0·28	0·33	0·37	0·43	0·51	0·56	0·62
60	0·10	0·13	0·16	0·20	0·24	0·29	0·34	0·38	0·45	0·52	0·58	0·65
2	0·10	0·13	0·16	0·20	0·25	0·30	0·35	0·40	0·46	0·53	0·60	0·67
4	0·11	0·14	0·17	0·21	0·26	0·31	0·37	0·41	0·47	0·55	0·62	0·69
6	0·11	0·14	0·17	0·22	0·27	0·32	0·38	0·43	0·49	0·56	0·64	0·71
8	0·12	0·15	0·18	0·23	0·28	0·33	0·39	0·44	0·51	0·58	0·66	0·73
70	0·12	0·15	0·19	0·24	0·29	0·34	0·40	0·46	0·53	0·60	0·68	0·76
2	0·12	0·15	0·19	0·24	0·29	0·35	0·41	0·48	0·54	0·61	0·70	0·78
4	0·12	0·16	0·20	0·25	0·30	0·36	0·42	0·50	0·55	0·63	0·72	0·80
6	0·13	0·16	0·21	0·26	0·31	0·37	0·43	0·51	0·57	0·65	0·74	0·82
8	0·13	0·17	0·21	0·26	0·31	0·38	0·44	0·52	0·59	0·67	0·76	0·84
80	0·13	0·17	0·22	0·27	0·33	0·39	0·45	0·53	0·61	0·69	0·78	0·87
2	0·14	0·17	0·22	0·28	0·33	0·40	0·46	0·54	0·62	0·70	0·80	0·89
4	0·14	0·18	0·23	0·29	0·35	0·41	0·47	0·55	0·63	0·72	0·82	0·91
6	0·14	0·18	0·23	0·29	0·35	0·42	0·48	0·56	0·65	0·74	0·84	0·93
8	0·15	0·19	0·24	0·30	0·37	0·43	0·49	0·57	0·67	0·76	0·86	0·95
90	0·15	0·20	0·25	0·31	0·37	0·44	0·51	0·59	0·68	0·78	0·88	0·98
2	0·15	0·20	0·25	0·31	0·38	0·45	0·52	0·60	0·69	0·79	0·90	1·00
4	0·16	0·21	0·26	0·32	0·39	0·46	0·53	0·61	0·70	0·81	0·92	1·02
6	0·16	0·21	0·26	0·33	0·40	0·47	0·54	0·62	0·72	0·83	0·94	1·04
8	0·17	0·22	0·27	0·34	0·41	0·48	0·55	0·64	0·74	0·85	0·96	1·06
100	0·17	0·22	0·27	0·34	0·41	0·49	0·57	0·66	0·76	0·87	0·98	1·09
2	0·17	0·22	0·27	0·34	0·42	0·50	0·58	0·67	0·77	0·88	0·99	1·11
4	0·17	0·23	0·28	0·35	0·42	0·50	0·59	0·68	0·78	0·90	1·01	1·13
6	0·18	0·23	0·28	0·35	0·43	0·51	0·60	0·69	0·80	0·92	1·03	1·15
8	0·18	0·24	0·29	0·36	0·44	0·52	0·61	0·71	0·81	0·94	1·05	1·17
110	0·18	0·24	0·29	0·37	0·45	0·53	0·63	0·73	0·83	0·95	1·07	1·19
	87° 40′	87° 20′	87° 0′	86° 40′	86° 20′	86° 0′	85° 40′	85° 20′	85° 0′	84° 40′	84° 20′	84° 0′
	2° 20′	2° 40′	3° 0′	3° 20′	3° 40′	4° 0′	4° 20′	4° 40′	5° 0′	5° 20′	5° 40′	6° 0′

Reduction of Distances.

83° 40′ 6° 20′	83° 20′ 6° 40′	83° 0′ 7° 0′	82° 40′ 7° 20′	82° 20′ 7° 40′	82° 0′ 8° 0′	81° 40′ 8° 20′	81° 20′ 8° 40′	81° 0′ 9° 0′	80° 40′ 9° 20′	80° 20′ 9° 40′	80° 0′ 10° 0′	
0·36	0·40	0·45	0·49	0·53	0·58	0·63	0·68	0·73	0·78	0·84	0·90	30
0·38	0·42	0·47	0·52	0·56	0·61	0·67	0·72	0·78	0·83	0·89	0·96	2
0·40	0·44	0·50	0·55	0·60	0·65	0·71	0·76	0·83	0·88	0·94	1·02	4
0·43	0·47	0·53	0·58	0·63	0·69	0·75	0·81	0·88	0·93	1·00	1·08	6
0·45	0·50	0·56	0·61	0·67	0·73	0·79	0·86	0·93	0·99	1·06	1·14	8
0·48	0·53	0·59	0·65	0·71	0·77	0·84	0·91	0·98	1·05	1·12	1·20	40
0·50	0·56	0·61	0·68	0·74	0·81	0·88	0·95	1·02	1·10	1·18	1·26	2
0·52	0·59	0·64	0·71	0·77	0·85	0·92	0·99	1·07	1·15	1·24	1·32	4
0·55	0·62	0·67	0·74	0·81	0·89	0·96	1·04	1·12	1·20	1·30	1·38	6
0·58	0·65	0·70	0·77	0·85	0·93	1·00	1·09	1·17	1·25	1·36	1·44	8
0·61	0·68	0·74	0·81	0·89	0·97	1·05	1·14	1·22	1·31	1·42	1·51	50
0·63	0·70	0·76	0·84	0·92	1·00	1·09	1·18	1·26	1·36	1·47	1·57	2
0·66	0·73	0·79	0·87	0·95	1·04	1·13	1·22	1·31	1·41	1·52	1·63	4
0·68	0·75	0·82	0·90	0·98	1·08	1·17	1·26	1·36	1·46	1·58	1·69	6
0·71	0·77	0·85	0·93	1·02	1·12	1·21	1·31	1·41	1·51	1·64	1·75	8
0·73	0·81	0·88	0·97	1·06	1·16	1·26	1·36	1·46	1·57	1·70	1·81	60
0·75	0·83	0·91	1·00	1·09	1·20	1·30	1·40	1·51	1·62	1·75	1·87	2
0·78	0·86	0·94	1·03	1·12	1·24	1·34	1·44	1·56	1·67	1·80	1·93	4
0·80	0·89	0·97	1·06	1·16	1·28	1·38	1·49	1·61	1·72	1·86	1·99	6
0·82	0·92	1·00	1·09	1·20	1·32	1·42	1·54	1·66	1·78	1·92	2·05	8
0·85	0·95	1·03	1·13	1·24	1·35	1·47	1·59	1·71	1·84	1·98	2·11	70
0·87	0·97	1·06	1·16	1·27	1·39	1·51	1·63	1·76	1·89	2·03	2·17	2
0·90	1·00	1·09	1·19	1·30	1·43	1·55	1·67	1·81	1·94	2·09	2·23	4
0·92	1·03	1·12	1·22	1·34	1·47	1·59	1·71	1·86	1·99	2·15	2·29	6
0·94	1·06	1·15	1·25	1·38	1·51	1·63	1·76	1·91	2·04	2·21	2·35	8
0·97	1·09	1·18	1·29	1·42	1·54	1·68	1·81	1·96	2·10	2·27	2·41	80
0·99	1·12	1·21	1·32	1·45	1·58	1·72	1·85	2·00	2·15	2·32	2·47	2
1·02	1·14	1·24	1·35	1·48	1·62	1·76	1·89	2·05	2·20	2·37	2·53	4
1·04	1·16	1·27	1·38	1·52	1·66	1·80	1·94	2·10	2·25	2·42	2·59	6
1·07	1·19	1·30	1·42	1·56	1·70	1·84	1·99	2·15	2·31	2·48	2·65	8
1·09	1·22	1·33	1·46	1·60	1·74	1·89	2·04	2·20	2·37	2·54	2·71	90
1·11	1·24	1·36	1·49	1·63	1·78	1·93	2·08	2·25	2·42	2·59	2·77	2
1·14	1·26	1·39	1·52	1·66	1·82	1·97	2·12	2·30	2·47	2·64	2·83	4
1·16	1·29	1·42	1·55	1·70	1·86	2·01	2·17	2·35	2·52	2·70	2·89	6
1·19	1·32	1·45	1·59	1·74	1·90	2·05	2·22	2·40	2·57	2·76	2·95	8
1·22	1·35	1·48	1·63	1·78	1·94	2·10	2·27	2·45	2·63	2·82	3·02	100
1·24	1·37	1·51	1·66	1·81	1·98	2·14	2·31	2·50	2·68	2·87	3·07	2
1·27	1·40	1·54	1·69	1·84	2·02	2·18	2·35	2·55	2·73	2·92	3·13	4
1·29	1·42	1·57	1·72	1·87	2·06	2·22	2·39	2·60	2·78	2·98	3·19	6
1·31	1·45	1·60	1·75	1·91	2·10	2·26	2·44	2·65	2·83	3·04	3·25	8
1·34	1·48	1·62	1·79	1·95	2·13	2·31	2·49	2·70	2·89	3·10	3·31	110
83° 40′ 6° 20′	83° 20′ 6° 40′	83° 0′ 7° 0′	82° 40′ 7° 20′	82° 20′ 7° 40′	82° 0′ 8° 0′	81° 40′ 8° 20′	81° 20′ 8° 40′	81° 0′ 9° 0′	80° 40′ 9° 20′	80° 20′ 9° 40′	80° 0′ 10° 0′	

I. (*cont.*)—TABLES FOR THE

	87° 40′ 2° 20′	87° 20′ 2° 40′	87° 0′ 3° 0′	86° 40′ 3° 20′	86° 20′ 3° 40′	86° 0′ 4° 0′	85° 40′ 4° 20′	85° 20′ 4° 40′	85° 0′ 5° 0′	84° 40′ 5° 20′	84° 20′ 5° 40′	84° 0′ 6° 0′
112	0·18	0·24	0·29	0·37	0·45	0·54	0·64	0·74	0·84	0·96	1·09	1·21
4	0·19	0·25	0·29	0·37	0·46	0·55	0·65	0·75	0·85	0·98	1·11	1·23
6	0·19	0·25	0·30	0·38	0·47	0·56	0·66	0·76	0·87	1·00	1·13	1·25
8	0·20	0·26	0·30	0·39	0·48	0·57	0·67	0·77	0·89	1·02	1·15	1·27
120	0·20	0·26	0·31	0·39	0·48	0·58	0·69	0·79	0·91	1·04	1·17	1·30
5	0·21	0·27	0·32	0·41	0·50	0·60	0·71	0·82	0·95	1·08	1·21	1·35
130	0·22	0·28	0·34	0·43	0·53	0·63	0·74	0·85	0·98	1·12	1·26	1·41
5	0·23	0·29	0·35	0·45	0·55	0·65	0·76	0·88	1·02	1·16	1·31	1·46
140	0·24	0·30	0·37	0·47	0·57	0·68	0·79	0·92	1·06	1·21	1·36	1·52
5	0·24	0·31	0·38	0·48	0·59	0·70	0·82	0·95	1·09	1·25	1·41	1·57
150	0·25	0·33	0·40	0·51	0·62	0·73	0·85	0·99	1·13	1·30	1·46	1·63
5	0·26	0·34	0·41	0·52	0·64	0·75	0·87	1·02	1·17	1·34	1·51	1·68
160	0·26	0·35	0·43	0·54	0·66	0·78	0·90	1·05	1·21	1·38	1·56	1·74
5	0·27	0·36	0·44	0·56	0·68	0·80	0·93	1·08	1·25	1·42	1·61	1·80
170	0·28	0·37	0·46	0·58	0·70	0·83	0·96	1·12	1·29	1·47	1·66	1·85
5	0·29	0·38	0·47	0·60	0·72	0·85	0·99	1·15	1·32	1·51	1·71	1·90
180	0·30	0·39	0·49	0·62	0·75	0·88	1·02	1·18	1·36	1·56	1·76	1·96
5	0·30	0·40	0·51	0·63	0·77	0·90	1·05	1·21	1·39	1·60	1·81	2·01
190	0·31	0·41	0·53	0·65	0·79	0·93	1·08	1·25	1·43	1·65	1·86	2·07
5	0·32	0·42	0·54	0·66	0·81	0·95	1·11	1·29	1·47	1·69	1·91	2·12
200	0·33	0·44	0·55	0·69	0·83	0·98	1·14	1·32	1·51	1·74	1·96	2·18
5	0·33	0·45	0·56	0·70	0·85	1·00	1·16	1·35	1·54	1·78	2·00	2·23
210	0·34	0·46	0·57	0·72	0·87	1·02	1·19	1·38	1·58	1·82	2·05	2·28
5	0·34	0·47	0·58	0·73	0·88	1·04	1·22	1·41	1·62	1·86	2·10	2·33
220	0·35	0·48	0·59	0·74	0·91	1·07	1·25	1·45	1·66	1·91	2·15	2·39
5	0·36	0·49	0·60	0·76	0·93	1·10	1·28	1·48	1·70	1·95	2·20	2·44
230	0·37	0·50	0·61	0·78	0·94	1·12	1·31	1·51	1·74	1·99	2·25	2·50
5	0·38	0·51	0·62	0·79	0·96	1·15	1·34	1·55	1·78	2·04	2·30	2·55
240	0·39	0·52	0·64	0·81	0·99	1·17	1·36	1·58	1·82	2·08	2·35	2·61
5	0·40	0·53	0·65	0·83	1·01	1·20	1·39	1·61	1·86	2·12	2·40	2·66
250	0·41	0·54	0·67	0·85	1·03	1·22	1·42	1·64	1·90	2·16	2·45	2·72
5	0·42	0·55	0·68	0·87	1·05	1·24	1·44	1·67	1·94	2·21	2·49	2·77
260	0·43	0·56	0·70	0·89	1·08	1·27	1·47	1·71	1·98	2·25	2·54	2·83
5	0·44	0·57	0·71	0·90	1·10	1·30	1·50	1·74	2·02	2·29	2·59	2·88
270	0·45	0·59	0·73	0·92	1·11	1·32	1·53	1·78	2·06	2·33	2·64	2·94
5	0·46	0·60	0·74	0·94	1·14	1·34	1·56	1·81	2·10	2·37	2·69	3·00
280	0·47	0·61	0·76	0·96	1·16	1·37	1·59	1·84	2·14	2·42	2·74	3·05
5	0·48	0·62	0·77	0·98	1·18	1·40	1·62	1·87	2·18	2·46	2·79	3·10
290	0·49	0·63	0·79	1·00	1·21	1·42	1·65	1·91	2·22	2·50	2·84	3·16
5	0·50	0·65	0·81	1·02	1·23	1·44	1·68	1·95	2·26	2·54	2·89	3·21
300	0·50	0·66	0·82	1·03	1·24	1·46	1·71	1·99	2·28	2·59	2·93	3·27
	87° 40′ 2° 20′	87° 20′ 2° 40′	87° 0′ 3° 0′	86° 40′ 3° 20′	86° 20′ 3° 40′	86° 0′ 4° 0′	85° 40′ 4° 20′	85° 20′ 4° 40′	85° 0′ 5° 0′	84° 40′ 5° 20′	84° 20′ 5° 40′	84° 0′ 6° 0

Reduction of Distances.

83° 40′ 6° 20′	83° 20′ 6° 40′	83° 0′ 7° 0′	82° 40′ 7° 20′	82° 20′ 7° 40′	82° 0′ 8° 0′	81° 40′ 8° 20′	81° 20′ 8° 40′	81° 0′ 9° 0′	80° 40′ 9° 20′	80° 20′ 9° 40′	80° 0′ 10° 0′	
1·36	1·50	1·65	1·82	1·98	2·16	2·35	2·53	2·74	2·94	3·15	3·37	112
1·39	1·53	1·68	1·85	2·01	2·20	2·39	2·57	2·79	2·99	3·20	3·43	4
1·41	1·56	1·71	1·88	2·05	2·24	2·43	2·62	2·84	3·04	3·26	3·49	6
1·44	1·59	1·74	1·91	2·09	2·28	2·47	2·67	2·89	3·09	3·32	3·55	8
1·46	1·62	1·77	1·95	2·13	2·32	2·52	2·72	2·94	3·15	3·38	3·61	120
1·52	1·68	1·84	2·03	2·22	2·42	2·65	2·83	3·06	3·28	3·52	3·76	5
1·58	1·75	1·92	2·12	2·31	2·52	2·73	2·95	3·19	3·41	3·66	3·91	130
1·64	1·82	2·00	2·20	2·40	2·62	2·83	3·06	3·31	3·54	3·80	4·06	5
1·70	1·89	2·07	2·28	2·50	2·72	2·94	3·18	3·43	3·67	3·94	4·21	140
1·76	1·95	2·14	2·36	2·58	2·82	3·04	3·29	3·55	3·81	4·08	4·37	5
1·82	2·02	2·22	2·45	2·67	2·91	3·15	3·40	3·67	3·95	4·23	4·52	150
1·88	2·08	2·29	2·53	2·75	3·00	3·25	3·51	3·79	4·08	4·37	4·67	5
1·94	2·15	2·37	2·61	2·84	3·10	3·36	3·62	3·92	4·21	4·51	4·82	160
2·00	2·22	2·46	2·62	2·93	3·20	3·46	3·74	4·04	4·34	4·65	4·97	5
2·06	2·29	2·52	2·77	3·02	3·30	3·57	3·85	4·14	4·47	4·79	5·12	170
2·12	2·36	2·59	2·85	3·11	3·40	3·67	3·96	4·27	4·60	4·93	5·27	5
2·19	2·43	2·67	2·93	3·20	3·49	3·78	4·08	4·39	4·73	5·07	5·43	180
2·25	2·49	2·74	3·01	3·29	3·59	3·88	4·19	4·51	4·85	5·21	5·58	5
2·31	2·56	2·81	3·09	3·38	3·69	3·99	4·31	4·64	4·98	5·35	5·73	190
2·37	2·63	2·88	3·17	3·47	3·78	4·09	4·42	4·77	5·12	5·49	5·88	5
2·44	2·70	2·96	3·26	3·56	3·88	4·20	4·54	4·90	5·26	5·64	6·03	200
2·50	2·76	3·03	3·34	3·64	3·97	4·30	4·65	5·02	5·39	5·78	6·18	5
2·56	2·83	3·11	3·42	3·73	4·07	4·41	4·76	5·14	5·52	5·92	6·33	210
2·62	2·91	3·18	3·50	3·82	4·16	4·51	4·87	5·26	5·65	6·06	6·48	5
2·68	2·97	3·26	3·58	3·91	4·26	4·62	4·98	5·39	5·78	6·20	6·63	220
2·73	3·03	3·33	3·66	4·00	4·35	4·72	5·09	5·51	5·91	6·34	6·78	5
2·80	3·10	3·41	3·75	4·09	4·45	4·83	5·21	5·63	6·04	6·48	6·93	230
2·86	3·17	3·48	3·83	4·18	4·54	4·93	5·32	5·75	6·17	6·62	7·08	5
2·92	3·24	3·56	3·91	4·27	4·64	5·04	5·44	5·88	6·30	6·76	7·23	240
2·98	3·30	3·63	3·99	4·36	4·73	5·14	5·55	6·00	6·43	6·90	7·38	5
3·04	3·37	3·71	4·07	4·45	4·83	5·25	5·67	6·12	6·56	7·04	7·53	250
3·10	3·44	3·78	4·15	4·54	4·92	5·35	5·78	6·24	6·69	7·18	7·68	5
3·16	3·51	3·86	4·24	4·63	5·02	5·46	5·90	6·37	6·82	7·32	7·83	260
3·22	3·57	3·93	4·32	4·72	5·11	5·56	6·01	6·49	6·95	7·46	7·98	5
3·28	3·64	4·01	4·40	4·81	5·21	5·67	6·13	6·61	7·09	7·60	8·13	270
3·34	3·71	4·08	4·48	4·90	5·30	5·77	6·24	6·73	7·22	7·76	8·28	5
3·40	3·78	4·16	4·56	4·99	5·41	5·88	6·36	6·86	7·35	7·88	8·43	280
3·46	3·84	4·23	4·64	5·08	5·50	5·98	6·47	6·98	7·48	8·02	8·58	5
3·52	3·91	4·31	4·72	5·17	5·61	6·09	6·59	7·10	7·62	8·17	8·73	290
3·58	3·97	4·38	4·80	5·25	5·70	6·19	6·70	7·22	7·75	8·31	8·88	5
3·65	4·04	4·46	4·89	5·34	5·81	6·30	6·81	7·34	7·89	8·46	9·04	300
83° 40′ 6° 20′	83° 20′ 6° 40′	83° 0′ 7° 0′	82° 40′ 7° 20′	82° 20′ 7° 40′	82° 0′ 8° 0′	81° 40′ 8° 20′	81° 20′ 8° 40′	81° 0′ 9° 0′	80° 40 9° 20′	80° 20′ 9° 40′	80° 0′ 9° 0′	

II.—TABLES FOR THE

	79° 40′	79° 20′	79° 0′	78° 40′	78° 20′	78° 0′	77° 40′	77° 20′	77° 0′	76° 40′	76° 20′	76° 0′
	10° 20′	10° 40′	11° 0′	11° 20′	11° 40′	12° 0′	12° 20′	12° 40′	13° 0′	13° 20′	13° 40′	14° 0′
30	0·96	1·02	1·09	1·16	1·23	1·30	1·37	1·44	1·52	1·59	1·67	1·75
2	1·03	1·08	1·16	1·24	1·31	1·38	1·46	1·54	1·62	1·70	1·78	1·84
4	1·09	1·15	1·23	1·32	1·39	1·47	1·54	1·63	1·72	1·80	1·89	1·96
6	1·16	1·22	1·30	1·39	1·47	1·55	1·63	1·73	1·82	1·91	2·00	2·07
8	1·21	1·29	1·37	1·46	1·55	1·64	1·72	1·83	1·92	2·02	2·11	2·19
40	1·28	1·36	1·45	1·54	1·63	1·72	1·82	1·92	2·02	2·12	2·23	2·33
2	1·34	1·43	1·52	1·62	1·71	1·81	1·91	2·02	2·12	2·23	2·34	2·42
4	1·40	1·50	1·59	1·70	1·79	1·89	2·00	2·11	2·22	2·33	2·45	2·54
6	1·46	1·57	1·67	1·77	1·88	1·98	2·09	2·21	2·32	2·44	2·57	2·66
8	1·53	1·64	1·74	1·85	1·94	2·07	2·19	2·30	2·42	2·55	2·68	2·78
50	1·60	1·71	1·82	1·93	2·04	2·16	2·28	2·40	2·53	2·66	2·79	2·92
2	1·66	1·77	1·89	2·01	2·12	2·24	2·37	2·49	2·63	2·76	2·90	3·04
4	1·72	1·84	1·96	2·09	2·20	2·33	2·46	2·58	2·73	2·87	3·01	3·15
6	1·78	1·91	2·04	2·16	2·28	2·41	2·55	2·68	2·83	2·97	3·12	3·27
8	1·85	1·98	2·11	2·24	2·36	2·50	2·64	2·78	2·93	3·08	3·23	3·39
60	1·92	2·05	2·18	2·31	2·44	2·59	2·73	2·88	3·03	3·19	3·34	3·50
2	1·98	2·11	2·25	2·39	2·52	2·67	2·82	2·97	3·13	3·29	3·45	3·62
4	2·04	2·18	2·32	2·47	2·60	2·76	2·91	3·07	3·23	3·40	3·56	3·73
6	2·10	2·25	2·40	2·55	2·68	2·84	3·00	3·16	3·33	3·50	3·67	3·85
8	2·17	2·32	2·48	2·63	2·76	2·93	3·09	3·26	3·43	3·61	3·78	3·96
70	2·24	2·39	2·55	2·70	2·85	3·02	3·18	3·36	3·54	3·72	3·90	4·08
2	2·30	2·45	2·64	2·78	2·93	3·10	3·27	3·45	3·64	3·82	4·01	4·20
4	2·36	2·52	2·69	2·86	3·01	3·19	3·36	3·55	3·74	3·93	4·12	4·31
6	2·42	2·59	2·77	2·94	3·09	3·27	3·45	3·65	3·84	4·03	4·23	4·43
8	2·49	2·66	2·84	3·01	3·17	3·36	3·54	3·74	3·94	4·14	4·34	4·54
80	2·56	2·73	2·91	3·08	3·25	3·45	3·63	3·84	4·04	4·25	4·46	4·65
2	2·62	2·79	2·98	3·16	3·33	3·53	3·72	3·93	4·14	4·35	4·57	4·77
4	2·68	2·86	3·06	3·24	3·42	3·62	3·81	4·02	4·24	4·46	4·68	4·89
6	2·74	2·93	3·14	3·32	3·50	3·70	3·90	4·12	4·34	4·57	4·79	5·01
8	2·81	3·00	3·21	3·40	3·58	3·79	3·99	4·22	4·45	4·68	4·90	5·13
90	2·88	3·08	3·28	3·47	3·66	3·88	4·09	4·32	4·55	4·79	5·02	5·25
2	2·94	3·16	3·35	3·55	3·74	3·97	4·18	4·41	4·65	4·89	5·13	5·36
4	3·00	3·22	3·42	3·63	3·82	4·05	4·27	4·51	4·75	5·00	5·24	5·48
6	3·07	3·29	3·48	3·71	3·91	4·14	4·36	4·60	4·85	5·10	5·35	5·60
8	3·14	3·34	3·57	3·79	3·99	4·23	4·46	4·71	4·95	5·21	5·47	5·72
100	3·21	3·41	3·64	3·86	4·08	4·32	4·56	4·81	5·06	5·32	5·58	5·85
2	3·27	3·47	3·71	3·94	4·16	4·40	4·65	4·90	5·16	5·42	5·69	5·97
4	3·33	3·54	3·78	4·02	4·24	4·49	4·74	5·00	5·26	5·53	5·80	6·09
6	3·39	3·61	3·86	4·10	4·32	4·57	4·83	5·10	5·36	5·63	5·91	6·21
8	3·46	3·68	3·93	4·17	4·40	4·66	4·92	5·20	5·46	5·74	6·02	6·33
110	3·53	3·75	4·00	4·24	4·49	4·75	5·01	5·29	5·56	5·85	6·13	6·45
	79° 40′	79° 20′	79° 0′	78° 40′	78° 20′	78° 0′	77° 40′	77° 20′	77° 0′	76° 40′	76° 20′	76° 0′
	10° 20′	10° 20′	11° 0′	11° 20′	11° 40′	12° 0′	12° 20′	12° 40′	13° 0′	13° 20′	13° 40′	14° 0′

Reduction of Distances.

75° 50′ 14° 10′	75° 40′ 14° 20′	75° 30′ 14° 30′	75° 20′ 14° 40′	75° 10′ 14° 50′	75° 0′ 15° 0′	74° 50′ 15° 10′	74° 40′ 15° 20′	74° 30′ 15° 30′	74° 20′ 15° 40′	74° 10′ 15° 50′	74° 0′ 16° 0	
1·79	1·83	1·88	1·92	1·96	2·01	2·05	2·09	2·14	2·18	2·23	2·28	30
1·91	1·95	2·00	2·04	2·09	2·14	2·19	2·22	2·28	2·33	2·38	2·43	2
2·03	2·08	2·13	2·17	2·22	2·27	2·32	2·37	2·43	2·48	2·53	2·58	4
2·15	2·20	2·25	2·30	2·35	2·40	2·46	2·51	2·57	2·63	2·68	2·73	6
2·27	2·32	2·38	2·43	2·48	2·53	2·59	2·65	2·71	2·77	2·83	2·88	8
2·39	2·45	2·51	2·56	2·62	2·68	2·73	2·79	2·85	2·91	2·97	3·04	40
2·51	2·57	3·63	2·69	2·75	2·81	2·87	2·93	2·99	3·05	3·12	3·19	2
2·63	2·69	2·76	2·81	2·88	2·94	3·00	3·07	3·13	3·20	3·27	3·34	4
2·75	2·81	2·88	2·94	3·01	3·07	3·14	3·21	3·27	3·34	3·42	3·49	6
2·87	2·94	3·01	3·08	3·14	3·21	3·28	3·35	3·41	3·48	3·57	3·64	8
3·00	3·06	3·14	3·20	3·27	3·35	3·42	3·50	3·57	3·65	3·72	3·80	50
3·12	3·18	3·26	3·32	3·40	3·48	3·56	3·64	3·71	3·79	3·87	3·95	2
3·24	3·30	3·39	3·45	3·53	3·62	3·69	3·78	3·86	3·94	4·02	4·10	4
3·36	3·43	3·51	3·58	3·66	3·75	3·83	3·92	4·00	4·08	4·16	4·25	6
3·48	3·55	3·64	3·71	3·79	3·89	3·97	4·06	4·14	4·22	4·31	4·40	8
3·59	3·67	3·76	3·84	3·92	4·02	4·10	4·20	4·28	4·37	4·46	4·56	60
3·71	3·79	3·89	3·96	4·05	4·16	4·24	4·34	4·43	4·52	4·62	4·71	2
3·83	3·91	4·01	4·09	4·18	4·29	4·37	4·48	4·57	4·66	4·77	4·86	4
3·95	4·03	4·14	4·23	4·31	4·43	4·51	4·62	4·71	4·81	4·91	5·01	6
4·07	4·16	4·26	4·36	4·44	4·55	4·65	4·76	4·85	4·95	5·05	5·16	8
4·19	4·28	4·39	4·48	4·58	4·69	4·79	4·90	5·00	5·10	5·20	5·32	70
4·31	4·40	4·51	4·60	4·71	4·82	4·92	5·04	5·14	5·24	5·34	5·47	2
4·43	4·52	4·64	4·73	4·84	4·96	5·06	5·18	5·29	5·39	5·39	5·62	4
4·55	4·64	4·76	4·86	4·97	5·09	5·19	5·32	5·43	5·53	5·54	5·77	6
4·62	4·76	4·89	4·99	5·10	5·22	5·33	5·46	5·57	5·68	5·69	5·92	8
4·79	4·89	5·01	5·12	5·23	5·36	5·47	5·60	5·72	5·83	5·95	6·08	80
4·91	5·01	5·14	5·24	5·36	5·49	5·60	5·74	5·85	5·97	6·10	6·23	2
5·03	5·14	5·26	5·37	5·49	5·62	5·74	5·88	5·99	6·12	6·25	6·38	4
5·15	5·26	5·39	5·50	5·62	5·76	5·88	6·02	6·13	6·26	6·40	6·53	6
5·27	5·38	5·51	5·63	5·75	5·89	6·02	6·16	6·27	6·41	6·55	6·68	8
5·39	5·51	5·64	5·76	5·89	6·03	6·16	6·30	6·43	6·56	6·70	6·84	90
5·51	5·63	5·76	5·89	6·02	6·16	6·30	6·44	6·57	6·70	6·84	6·99	2
5·63	5·76	5·89	6·02	6·15	6·30	6·44	6·58	6·71	6·85	6·99	7·14	4
5·75	5·88	6·01	6·15	6·28	6·43	6·58	6·72	6·85	6·99	7·14	7·29	6
5·87	6·00	6·14	6·28	9·41	6·56	6·72	6·86	6·99	7·14	7·29	7·44	8
5·99	6·13	6·27	6·41	6·55	6·70	6·85	6·99	7·14	7·29	7·44	7·60	100
6·11	6·25	6·39	6·54	6·68	6·83	6·99	7·13	7·29	7·43	7·59	7·75	2
6·23	6·37	6·51	6·56	6·81	6·97	7·12	7·27	7·43	7·58	7·74	7·90	4
6·35	6·49	6·64	6·69	6·94	7·01	7·26	7·41	7·57	7·72	7·89	8·c5	6
6·47	6·61	6·77	6·92	7·07	7·24	7·40	7·55	7·71	7·87	8·04	8·20	8
6·59	6·74	6·90	7·05	7·21	7·37	7·53	7·69	7·85	8·01	8·18	8·35	110
75° 50′ 14° 10′	75° 40′ 14° 20′	75° 30′ 14° 30′	75° 20′ 14° 40′	75° 10′ 14° 50′	75° 0′ 15° 0′	74° 50′ 15° 10′	74° 40′ 15° 20′	74° 30′ 15° 30′	74° 20 15° 40	74° 10′ 15° 50′	74° 0 16° 0′	

II. (*cont.*)—TABLES FOR THE

	79° 40′	79° 20′	79° 0′	78° 40′	78° 20′	78° 0′	77° 40′	77° 20′	77° 0′	76° 40′	76° 20′	76° 0′
	10° 20′	10° 40′	11° 0′	11° 20′	11° 40′	12° 0′	12° 20′	12° 40′	13° 0′	13° 20′	13° 40′	14° 0′
112	3·59	3·81	4·07	4·31	4·57	4·83	5·10	5·38	5·66	5·95	6·24	6·57
4	3·65	3·88	4·14	4·39	4·65	4·92	5·19	5·47	5·76	6·06	6·35	6·68
6	3·71	3·95	4·21	4·47	4·73	5·01	5·28	5·56	5·86	6·16	6·47	6·80
8	3·78	4·02	4·28	4·54	4·81	5·09	5·37	5·64	5·96	6·27	6·58	6·91
120	3·85	4·11	4·35	4·62	4·90	5·18	5·46	5·72	6·07	6·38	6·70	7·03
5	4·01	4·26	4·54	4·82	5·11	5·40	5·70	6·01	6·32	6·65	6·97	7·31
130	4·17	4·43	4·72	5·01	5·31	5·61	5·92	6·25	6·57	6·91	7·25	7·60
5	4·33	4·60	4·90	5·21	5·52	5·83	6·15	6·49	6·83	7·17	7·53	7·90
140	4·49	4·77	5·09	5·40	5·73	6·05	6·38	6·73	7·08	7·44	7·81	8·19
5	4·65	4·94	5·27	5·60	5·93	6·27	6·61	6·97	7·33	7·70	8·09	8·49
150	4·82	5·14	5·46	5·80	6·13	6·48	6·85	7·21	7·59	7·98	8·37	8·78
5	4·98	5·30	5·64	5·99	6·33	6·69	7·08	7·45	7·84	8·24	8·65	9·07
160	5·14	5·48	5·82	6·18	6·54	6·91	7·31	7·69	8·10	8·51	8·93	9·37
5	5·30	5·64	6·00	6·38	6·74	7·12	7·55	7·93	8·35	8·77	9·21	9·66
170	5·46	5·81	6·18	6·57	6·95	7·34	7·78	8·17	8·61	9·03	9·49	9·95
5	5·62	5·98	6·36	6·76	7·15	7·56	8·00	8·41	8·86	9·30	9·77	10·25
180	5·78	6·16	6·54	6·95	7·36	7·78	8·22	8·66	9·11	9·57	10·04	10·54
5	5·94	6·33	6·72	7·15	7·56	8·00	8·45	8·90	9·36	9·84	10·32	10·83
190	6·10	6·51	6·90	7·34	7·77	8·22	8·67	9·14	9·62	10·11	10·60	11·13
5	6·26	6·66	7·08	7·53	7·97	8·43	8·90	9·38	9·87	10·37	10·88	11·42
200	6·42	6·85	7·26	7·72	8·18	8·65	9·12	9·62	10·12	10·64	11·16	11·72
5	6·58	7·03	7·44	7·91	8·38	8·86	9·34	9·86	10·37	10·90	11·43	12·01
210	6·74	7·20	7·62	8·10	8·58	9·08	9·57	10·10	10·62	11·17	11·71	12·30
5	6·90	7·36	7·80	8·29	8·78	9·29	9·79	10·34	10·87	11·43	11·98	12·59
220	7·06	7·54	7·99	8·48	8·99	9·51	10·02	10·58	11·13	11·70	12·27	12·89
5	7·22	7·70	8·17	8·67	9·19	9·72	10·24	10·82	11·38	11·96	12·55	13·18
230	7·38	7·88	8·35	8·86	9·40	9·94	10·47	11·06	11·63	12·23	12·83	13·47
5	7·54	8·04	8·51	9·05	9·60	10·15	10·69	11·30	11·88	12·49	13·10	13·76
240	7·70	8·22	8·71	9·25	9·81	10·37	10·93	11·54	12·14	12·76	13·39	14·06
5	7·86	8·39	8·79	9·44	10·01	10·58	11·16	11·78	12·39	13·02	13·67	14·35
250	8·02	8·55	9·08	9·64	10·22	10·80	11·39	12·02	12·65	13·29	13·95	14·63
5	8·18	8·73	9·26	9·83	10·42	11·01	11·61	12·26	12·90	13·55	14·23	14·92
260	8·34	8·89	9·44	10·03	10·63	11·23	11·85	12·50	13·15	13·82	14·51	15·21
5	8·50	9·06	9·62	10·22	10·83	11·44	12·08	12·74	13·40	14·08	14·79	15·50
270	8·66	9·24	9·81	10·42	11·04	11·66	12·31	12·98	13·66	14·35	15·07	15·80
5	8·82	9·41	9·99	10·61	11·24	11·87	12·54	13·22	13·91	14·61	15·35	16·09
280	8·98	9·59	10·18	10·81	11·45	12·10	12·77	13·46	14·16	14·88	15·63	16·38
5	9·14	9·76	10·36	11·00	11·65	12·31	13·00	13·70	14·41	15·14	15·91	16·67
290	9·31	9·94	10·55	11·20	11·86	12·54	13·23	13·94	14·67	15·41	16·19	16·97
5	9·47	10·11	10·73	11·39	12·06	12·75	13·46	14·18	14·92	15·87	16·47	17·26
300	9·64	10·28	10·92	11·59	12·27	12·97	13·69	14·43	15·18	15·95	16·74	17·56
	79° 40′	79° 20′	79° 0′	78° 40′	78° 20′	78° 0′	77° 40′	77° 20′	77° 0′	76° 40′	76° 20′	76° 0′
	10° 20′	10° 40′	11° 0′	11° 20′	11° 40′	12° 0′	12° 20′	12° 40′	13° 0′	13° 20′	13° 40′	14° 0′

REDUCTION OF DISTANCES.

75° 50′ 14° 10′	75° 40′ 14° 20′	75° 30′ 14° 30′	75° 20′ 14° 40′	75° 10′ 14° 50′	75° 0′ 15° 0′	74° 50′ 15° 10′	74° 40′ 15° 20′	74° 30′ 15° 30′	74° 20′ 15° 40′	74° 10′ 15° 50′	74° 0′ 16° 0′	
6·71	6·86	7·02	7·18	7·34	7·50	7·67	7·83	7·99	8·16	8·33	8·50	112
6·83	6·98	7·15	7·31	7·47	7·64	7·81	7·97	8·14	8·30	8·47	8·65	4
6·95	7·10	7·27	7·44	7·60	7·77	7·95	8·11	8·28	8·45	8·62	8·80	6
7·07	7·22	7·39	7·57	7·73	7·90	8·09	8·25	8·43	8·60	8·77	8·95	8
7·19	7·35	7·52	7·70	7·86	8·04	8·21	8·39	8·57	8·75	8·93	9·11	120
7·49	7·56	7·83	8·01	8·18	8·38	8·55	8·74	8·93	9·11	9·30	9·50	5
7·79	7·97	8·15	8·33	8·51	8·71	8·89	9·09	9·28	9·47	9·67	9·87	130
8·09	8·28	8·46	8·65	8·84	9·04	9·23	9·44	9·65	9·84	10·04	10·25	5
8·39	8·58	8·78	8·97	9·17	9·38	9·57	9·79	10·00	10·20	10·42	10·63	140
8·69	8·88	9·09	9·30	9·50	9·71	9·92	10·14	10·35	10·57	10·79	11·01	5
8·99	9·19	9·40	9·62	9·82	10·05	10·27	10·49	10·71	10·93	11·16	11·40	150
9·29	9·50	9·71	9·94	10·14	10·38	10·61	10·84	11·07	11·29	11·53	11·78	5
9·59	9·80	10·03	10·26	10·47	10·72	10·95	11·19	11·43	11·66	11·90	12·16	160
9·89	10·11	10·34	10·58	10·79	11·05	11·30	11·54	11·78	12·02	12·28	12·54	5
10·29	10·41	10·66	10·90	11·12	11·38	11·64	11·89	12·14	12·39	12·65	12·92	170
10·59	10·72	10·98	11·22	11·45	11·72	11·98	12·24	12·50	12·75	13·02	13·30	5
10·78	11·04	11·29	11·54	11·78	12·06	12·32	12·59	12·87	13·12	13·39	13·68	180
11·08	11·34	11·60	11·86	12·11	12·39	12·66	12·93	13·22	13·48	13·76	14·06	5
11·38	11·65	11·92	12·18	12·44	12·73	13·01	13·28	13·57	13·85	14·14	14·44	190
11·68	11·95	12·23	12·50	12·77	13·06	13·35	13·62	13·93	14·22	14·51	14·81	5
11·98	12·26	12·55	12·82	13·10	13·40	13·70	13·98	14·28	14·58	14·88	15·19	200
12·28	12·56	12·86	13·14	13·43	13·73	14·04	14·33	14·63	14·94	15·25	15·57	5
12·58	12·87	13·17	13·46	13·75	14·07	14·38	14·68	14·99	15·30	15·62	15·95	210
12·88	13·17	13·48	13·78	14·08	14·40	14·72	15·03	15·35	15·66	15·99	16·33	5
13·18	13·48	13·80	14·10	14·41	14·74	15·07	15·38	15·71	16·03	16·37	16·71	220
13·48	13·79	14·11	14·42	14·74	15·07	15·41	15·73	16·07	16·39	11·74	17·09	5
13·78	14·10	14·42	14·74	15·06	15·41	15·75	16·08	16·43	16·76	17·11	17·47	230
14·08	14·40	14·73	15·06	15·39	15·74	16·09	16·43	16·79	17·12	17·48	17·85	5
14·38	14·71	15·05	15·39	15·72	16·08	16·42	16·78	17·14	17·49	17·86	18·23	240
14·68	15·01	15·36	15·71	16·05	16·41	16·76	17·13	17·50	17·85	18·23	18·61	5
14·98	15·32	15·67	16·03	16·37	16·75	17·10	17·48	17·86	18·22	18·60	18·99	250
15·28	15·63	15·98	16·35	16·70	17·08	17·44	17·83	18·22	18·58	18·97	19·37	5
15·58	15·94	16·30	16·67	17·03	17·42	17·78	18·18	18·57	18·95	19·35	19·75	260
15·88	16·24	16·61	16·99	17·36	17·75	18·12	18·53	18·93	19·31	19·72	20·13	5
16·18	16·55	16·92	17·31	17·68	18·09	18·46	18·88	19·29	19·68	20·09	20·51	270
16·48	16·85	17·23	17·63	18·01	18·42	18·80	19·23	19·65	20·04	20·46	20·89	5
16·78	17·16	17·55	17·95	18·34	18·76	19·15	19·58	20·00	20·41	20·84	21·27	280
17·08	17·47	17·86	18·27	18·67	19·09	19·49	19·93	20·35	20·77	21·21	21·65	5
17·38	17·77	18·18	18·60	18·99	19·43	19·84	20·28	20·71	21·14	21·58	22·03	290
17·67	18·08	18·49	18·92	19·32	19·76	20·19	20·63	21·07	21·50	21·95	22·41	5
17·96	18·38	18·81	19·24	19·65	20·10	20·54	20·98	21·43	21·87	22·33	22·79	300
75° 50′ 14° 10′	75° 40′ 14° 20′	75° 30′ 14° 30′	75° 20′ 14° 40′	75° 10′ 14° 50′	75° 0′ 15° 0′	74° 50′ 15° 10′	74° 40′ 15° 20′	74° 30′ 15° 30′	74° 20′ 15° 40′	74° 10′ 15° 50′	74° 0′ 16° 0′	

III.—Tables for the

	73° 50′ 16° 10′	73° 40′ 16° 20′	73° 30′ 16° 30′	73° 20′ 16° 40′	73° 10′ 16° 50′	73° 0′ 17° 0′	72° 50′ 17° 10′	72° 40′ 17° 20′	72° 30′ 17° 30′	72° 20′ 17° 40′	72° 10′ 17° 50′	72° 0′ 18° 0′
30	2·32	2·37	2·42	2·47	2·52	2·56	2·61	2·66	2·71	2·76	2·81	2·86
2	2·47	2·53	2·58	2·63	2·69	2·73	2·78	2·82	2·88	2·94	3·00	3·05
4	2·63	2·69	2·74	2·80	2·85	2·90	2·96	3·00	3·06	3·13	3·18	3·24
6	2·78	2·84	2·90	2·96	3·02	3·07	3·13	3·17	3·23	3·31	3·37	3·43
8	2·94	3·00	3·06	3·12	3·18	3·24	3·30	3·35	3·42	3·50	3·56	3·62
40	3·10	3·16	3·22	3·29	3·35	3·41	3·48	3·53	3·60	3·68	3·74	3·81
2	3·25	3·32	3·38	3·45	3·51	3·58	3·65	3·71	3·79	3·87	3·93	4·00
4	3·41	3·48	3·54	3·61	3·68	3·75	3·82	3·88	3·97	4·05	4·12	4·19
6	3·56	3·63	3·70	3·78	3·85	3·92	4·00	4·06	4·15	4·24	4·31	4·38
8	3·72	3·79	3·86	3·94	4·02	4·09	4·17	4·24	4·33	4·42	4·49	4·57
50	3·87	3·95	4·03	4·11	4·19	4·27	4·35	4·43	4·52	4·61	4·69	4·77
2	4·03	4·11	4·19	4·27	4·36	4·44	4·52	4·61	4·70	4·79	4·88	4·96
4	4·18	4·27	4·35	4·43	4·52	4·61	4·70	4·79	4·88	4·97	5·07	5·15
6	4·34	4·43	4·51	4·60	4·69	4·78	4·87	4·96	5·06	5·16	5·26	5·34
8	4·50	4·58	4·67	4·76	4·86	4·95	5·05	5·14	5·24	5·34	5·45	5·53
60	4·65	4·74	4·84	4·94	5·03	5·12	5·22	5·32	5·42	5·52	5·63	5·73
2	4·80	4·90	5·00	5·10	5·20	5·29	5·39	5·49	5·60	5·71	5·82	5·92
4	4·96	5·05	5·16	5·26	5·36	5·46	5·57	5·67	5·78	5·89	6·01	6·11
6	5·11	5·21	5·32	5·43	5·53	5·63	5·74	5·85	5·96	6·08	6·20	6·30
8	5·27	5·37	5·48	5·59	5·70	5·80	5·91	6·02	6·14	6·26	6·39	6·49
70	5·42	5·53	5·65	5·76	5·87	5·98	6·09	6·21	6·33	6·45	6·57	6·68
2	5·58	5·69	5·81	5·92	6·03	6·15	6·26	6·38	6·51	6·63	6·76	6·87
4	5·74	5·85	5·97	6·08	6·20	6·32	6·43	6·56	6·69	6·82	6·95	7·06
6	5·89	6·00	6·13	6·24	6·37	6·49	6·60	6·73	6·87	7·00	7·14	7·25
8	6·05	6·16	6·29	6·42	6·54	6·66	6·77	6·91	7·05	7·18	7·32	7·44
80	6·20	6·32	6·45	6·59	6·71	6·83	6·96	7·09	7·23	7·37	7·50	7·64
2	6·35	6·48	6·61	6·75	6·88	7·00	7·13	7·27	7·41	7·55	7·69	7·83
4	6·51	6·64	6·77	6·91	7·04	7·17	7·30	7·45	7·59	7·73	7·88	8·02
6	6·66	6·80	6·93	7·07	7·21	7·34	7·47	7·62	7·77	7·92	8·07	8·21
8	6·81	6·95	7·09	7·24	7·38	7·52	7·65	7·80	7·95	8·10	8·25	8·40
90	6·97	7·11	7·25	7·41	7·55	7·70	7·83	7·98	8·14	8·29	8·44	8·60
2	7·12	7·17	7·41	7·57	7·72	7·87	8·00	8·16	8·32	8·48	8·63	8·79
4	7·28	7·33	7·57	7·73	7·89	8·04	8·18	8·34	8·50	8·66	8·82	8·98
6	7·43	7·49	7·73	7·89	8·06	8·21	8·35	8·51	8·68	8·84	9·01	9·17
8	7·59	7·64	7·89	8·06	8·23	8·38	8·53	8·69	8·86	9·03	9·20	9·36
100	7·75	7·91	8·06	8·23	8·39	8·55	8·71	8·88	9·04	9·21	9·38	9·55
2	7·90	8·05	8·22	8·39	8·56	8·72	8·88	9·06	9·22	9·40	9·57	9·74
4	8·06	8·21	8·38	8·55	8·73	8·89	9·05	9·25	9·40	9·58	9·76	9·93
6	8·21	8·37	8·54	8·72	8·90	9·06	9·23	9·43	9·58	9·76	9·95	10·12
8	8·36	8·53	8·70	8·89	9·07	9·23	9·40	9·61	9·76	9·95	10·14	10·31
110	8·52	8·70	8·86	9·05	9·23	9·40	9·58	9·78	9·94	10·13	10·32	10·50
	73° 50′ 16° 10′	73° 40′ 16° 20	73° 30′ 16° 30′	73° 20′ 16° 40′	73° 10′ 16° 50′	73° 0′ 17° 0′	72° 50′ 17° 10′	72° 40′ 17° 20′	72° 30′ 17° 30′	72° 20′ 17° 40′	72° 10′ 17° 50′	72° 0′ 18° 0′

Reduction of Distances.

71° 50′ 18° 10′	71° 40′ 18° 20′	71° 30′ 18° 30′	71° 20′ 18° 40′	71° 10′ 18° 50′	71° 0′ 19° 0′	70° 50′ 19° 10′	70° 40′ 19° 20′	70° 30′ 19° 30′	70° 20′ 19° 40′	70° 10′ 19° 50′	70° 0′ 20° 0′	
2·91	2·97	3·02	3·07	3·13	3·18	3·23	3·29	3·34	3·40	3·45	3·51	30
3·10	3·17	3·22	3·27	3·34	3·39	3·44	3·51	3·56	3·63	3·63	3·74	2
3·30	3·36	3·42	3·48	3·55	3·60	3·66	3·73	3·78	3·85	3·91	3·98	4
3·49	3·56	3·62	3·68	3·75	3·81	3·87	3·95	4·01	4·08	4·14	4·21	6
3·69	3·76	3·82	3·89	3·96	4·02	4·09	4·17	4·23	4·31	4·37	4·44	8
3·88	3·95	4·03	4·09	4·17	4·23	4·31	4·39	4·45	4·53	4·60	4·68	40
4·07	4·15	4·23	4·30	4·37	4·44	4·52	4·61	4·68	4·76	4·83	4·91	2
4·27	4·35	4·43	4·50	4·58	4·65	4·73	4·83	4·90	4·98	5·06	5·14	4
4·46	4·54	4·63	4·71	4·79	4·86	4·95	5·05	5·12	5·21	5·29	5·38	6
4·66	4·74	4·83	4·91	5·00	5·08	5·17	5·27	5·34	5·43	5·52	5·61	8
4·86	4·94	5·03	5·12	5·21	5·29	5·39	5·48	5·57	5·66	5·75	5·85	50
5·05	5·13	5·23	5·32	5·42	5·50	5·60	5·70	5·79	5·89	5·98	6·08	2
5·25	5·33	5·43	5·53	5·63	5·71	5·82	5·92	6·01	6·12	6·21	6·31	4
5·44	5·52	5·63	5·73	5·83	5·92	6·03	6·14	6·24	6·34	6·44	6·55	6
5·64	5·72	5·83	5·94	6·04	6·13	6·25	6·36	6·46	6·57	6·67	6·78	8
5·83	5·92	6·04	6·14	6·25	6·34	6·46	6·58	6·68	6·79	6·90	7·01	60
6·03	6·11	6·24	6·35	6·46	6·55	6·68	6·80	6·91	7·02	7·13	7·25	2
6·22	6·31	6·44	6·55	6·67	6·77	6·89	7·02	7·13	7·24	7·36	7·48	4
6·42	6·51	6·64	6·76	6·88	6·98	7·11	7·24	7·35	7·47	7·59	7·71	6
6·61	6·71	6·84	6·96	7·09	7·20	7·32	7·46	7·57	7·70	7·82	7·94	8
6·80	6·92	7·05	7·17	7·30	7·42	7·54	7·67	7·80	7·92	8·05	8·18	70
7·00	7·12	7·25	7·37	7·50	7·63	7·75	7·89	8·02	8·15	8·28	8·41	2
7·19	7·31	7·45	7·58	7·71	7·84	7·97	8·11	8·24	8·38	8·51	8·64	4
7·39	7·51	7·65	7·79	7·92	8·05	8·18	8·33	8·47	8·60	8·74	8·88	6
7·58	7·70	7·85	8·00	8·13	8·26	8·42	8·55	8·69	8·83	8·91	9·11	8
7·78	7·89	8·05	8·20	8·34	8·47	8·63	8·77	8·91	9·05	9·20	9·34	80
7·97	8·09	8·25	8·41	8·55	8·68	8·85	8·99	9·14	9·28	9·43	9·58	2
8·17	8·29	8·45	8·61	8·76	8·89	9·06	9·21	9·36	9·50	9·66	9·81	4
8·36	8·49	8·65	8·82	8·97	9·10	9·28	9·43	9·58	9·73	9·89	10·04	6
8·56	8·69	8·85	9·02	9·17	9·32	9·49	9·65	9·80	9·96	10·12	10·28	8
8·75	8·90	9·06	9·22	9·38	9·54	9·70	9·96	10·03	10·19	10·36	10·53	90
8·95	9·10	9·26	9·43	9·59	9·75	9·92	10·08	10·25	10·41	10·59	10·75	2
9·14	9·29	9·46	9·63	9·80	9·96	10·13	10·30	10·47	10·64	10·82	10·98	4
9·34	9·49	9·66	9·83	10·01	10·17	10·35	10·52	10·69	10·87	11·05	11·22	6
9·53	9·69	9·86	10·03	10·21	10·38	10·56	10·74	10·92	11·10	11·28	11·45	8
9·72	9·89	10·07	10·24	10·42	10·60	10·78	10·96	11·14	11·32	11·51	11·69	100
9·92	10·09	10·27	10·44	10·63	10·81	10·99	11·17	11·36	11·55	11·74	11·92	2
10·11	10·29	10·47	10·65	10·84	11·02	11·21	11·39	11·58	11·75	11·97	12·16	4
10·31	10·49	10·67	10·85	11·05	11·23	11·43	11·61	11·81	12·00	12·20	12·39	6
10·50	10·69	10·87	11·06	11·26	11·44	11·64	11·83	12·03	12·23	12·43	12·63	8
10·69	10·88	11·08	11·26	11·46	11·65	11·85	12·05	12·25	12·45	12·66	12·86	110
71° 50′ 18° 10′	71° 40′ 18° 20′	71° 30′ 18° 30′	71° 20′ 18° 40′	71° 10′ 18° 50′	71° 0′ 19° 0′	70° 50′ 19° 10′	70° 40′ 19° 20′	70° 30′ 19° 30′	70° 20′ 19° 40′	70° 10′ 19° 50′	70° 0′ 20° 0′	

III. (*cont.*)—TABLES FOR THE

	73° 50′ 16° 10′	73° 40′ 16° 20′	73° 30′ 16° 30′	73° 20′ 16° 40′	73° 10′ 16° 50′	73° 0′ 17° 0′	72° 50′ 17° 10′	72° 40′ 17° 20′	72° 30′ 17° 30′	72° 20′ 17° 40′	72° 10′ 17° 50′	72° 0′ 18° 0′
112	8·67	8·86	9·02	9·22	9·40	9·57	9·75	9·96	10·12	10·30	10·51	10·69
4	8·83	9·01	9·18	9·38	9·57	9·74	9·93	10·13	10·30	10·50	10·70	10·88
6	8·98	9·17	9·34	9·55	9·74	9·91	10·10	10·30	10·48	10·68	10·89	11·07
8	9·14	9·33	9·50	9·71	9·90	10·08	10·27	10·47	10·66	10·87	11·08	11·26
120	9·30	9·49	9·67	9·87	10·06	10·26	10·45	10·65	10·85	11·05	11·26	11·46
5	9·69	9·89	10·07	10·28	10·53	10·69	10·88	11·10	11·30	11·51	11·73	11·93
130	10·07	10·28	10·47	10·69	10·90	11·11	11·32	11·54	11·75	11·97	12·19	12·40
5	10·47	10·68	10·87	11·10	11·37	11·54	11·75	11·99	12·20	12·43	12·66	12·87
140	10·85	11·07	11·28	11·51	11·73	11·96	12·19	12·43	12·66	12·89	13·13	13·36
5	11·24	11·47	11·68	11·92	12·15	12·39	12·62	12·88	13·11	13·35	13·60	13·84
150	11·62	11·86	12·10	12·34	12·58	12·82	13·06	13·31	13·56	13·81	14·07	14·32
5	12·00	12·26	12·50	12·75	13·00	13·24	13·49	13·75	14·01	14·27	14·54	14·79
160	12·39	12·64	12·90	13·16	13·42	13·67	13·93	14·20	14·46	14·73	15·01	15·27
5	12·79	13·04	13·30	13·57	13·83	14·09	14·36	14·65	14·91	15·19	15·48	15·74
170	13·17	13·44	13·71	13·98	14·25	14·51	14·80	15·09	15·37	15·65	15·95	16·22
5	13·56	13·84	14·11	14·39	14·67	14·94	15·23	15·54	15·82	16·11	16·42	16·69
180	13·94	14·22	14·51	14·80	15·09	15·36	15·67	15·98	16·27	16·57	16·88	17·17
5	14·33	14·62	14·91	15·21	15·51	15·79	16·10	16·43	16·72	17·03	17·35	17·64
190	14·72	15·02	15·32	15·62	15·93	16·22	16·54	16·87	17·18	17·49	17·82	18·12
5	15·11	15·42	15·72	16·03	16·35	16·65	16·97	17·31	17·63	17·95	18·29	18·60
200	15·50	15·82	16·12	16·45	16·77	17·09	17·42	17·76	18·08	18·42	18·76	19·09
5	15·88	16·22	16·52	16·86	17·19	17·54	17·85	18·21	18·53	18·88	19·23	19·57
210	16·27	16·61	16·93	17·27	17·60	17·94	18·29	18·65	18·98	19·34	19·70	20·05
5	16·66	16·81	17·33	17·68	18·02	18·37	18·72	19·10	19·43	19·80	20·17	20·52
220	17·05	17·40	17·73	18·09	18·44	18·80	19·16	19·53	19·89	20·26	20·64	21·00
5	17·44	17·80	18·14	18·50	18·86	19·22	19·59	19·98	20·34	20·72	21·11	21·47
230	17·82	18·19	18·54	18·92	19·28	19·65	20·03	20·41	20·79	21·18	21·57	21·95
5	18·21	18·59	18·94	19·33	19·70	20·07	20·46	20·86	21·24	21·64	22·04	22·43
240	18·60	18·98	19·35	19·74	20·12	20·51	20·90	21·30	21·70	22·10	22·51	22·91
5	18·99	19·38	19·75	20·15	20·52	20·93	21·33	21·75	22·15	22·56	22·98	23·39
250	19·38	19·77	20·15	20·56	20·96	21·36	21·77	22·20	22·60	23·02	23·45	23·86
5	19·77	20·16	20·46	20·97	21·38	21·79	22·20	22·64	23·05	23·48	23·92	24·34
260	20·15	20·56	20·96	21·38	21·80	22·22	22·64	23·08	23·51	23·94	24·38	24·81
5	20·54	20·96	21·36	21·79	22·22	22·64	23·07	23·53	23·96	24·40	24·85	25·29
270	20·93	21·35	21·76	22·21	22·64	23·07	23·51	23·97	24·41	24·86	25·32	25·76
5	21·32	21·75	22·16	22·62	23·06	23·49	23·94	24·42	24·86	25·32	25·79	26·24
280	21·70	22·14	22·57	23·04	23·48	23·92	24·38	24·85	25·32	25·78	26·25	26·72
5	22·09	22·54	22·97	23·45	23·90	24·35	24·81	25·30	25·77	26·24	26·72	27·20
290	22·48	22·93	23·38	23·86	24·32	24·78	25·25	25·74	26·22	26·70	27·19	27·68
5	22·87	23·33	23·78	24·27	24·74	25·21	25·68	26·19	26·67	27·16	27·66	28·16
300	23·25	23·72	24·19	24·68	25·16	25·64	26·13	26·63	27·12	27·62	28·14	28·65
	73° 50′ 16° 10′	73° 40′ 16° 20′	73° 30′ 16° 30′	73° 20′ 16° 40′	73° 10′ 16° 50′	73° 0′ 17° 0′	72° 50′ 17° 10′	72° 40′ 17° 20′	72° 30′ 17° 30′	72° 20′ 17° 40′	72° 10′ 17° 50′	72° 0′ 18° 0′

REDUCTION OF DISTANCES.

71° 50′ 18° 10′	71° 40′ 18° 20′	71° 30′ 18° 30′	71° 20′ 18° 40′	71° 10′ 18° 50′	71° 0′ 19° 0′	70° 50′ 19° 10′	70° 40′ 19° 20′	70° 30′ 19° 30′	70° 20′ 19° 40′	70° 10′ 19° 50′	70° 0′ 20° 0′	
10·89	11·08	11·28	11·47	11·67	11·86	12·07	12·27	12·47	12·68	12·89	13·10	112
11·08	11·28	11·48	11·67	11·87	12·07	12·28	12·49	12·69	12·91	13·12	13·33	4
11·28	11·48	11·68	11·88	12·08	12·28	12·50	12·71	12·92	13·14	13·35	13·57	6
11·47	11·68	11·88	12·08	12·29	12·49	12·71	12·93	13·14	13·37	13·58	13·80	8
11·66	11·87	12·08	12·29	12·50	12·71	12·93	13·15	13·37	13·59	13·81	14·03	120
12·14	12·37	12·58	12·80	13·02	13·24	13·47	13·69	13·93	14·15	14·38	14·61	5
12·62	12·86	13·08	13·32	13·55	13·78	14·01	14·25	14·48	14·77	14·96	15·20	130
13·10	13·36	13·58	13·83	14·07	14·31	14·55	14·79	15·04	15·28	15·53	15·78	5
13·61	13·85	14·09	14·34	14·59	14·84	15·09	15·34	15·59	15·84	16·11	16·37	140
14·09	14·35	14·59	14·86	15·11	15·37	15·63	15·89	16·15	16·40	16·68	16·95	5
14·57	14·84	15·10	15·36	15·63	15·90	16·17	16·44	16·71	16·99	17·27	17·54	150
15·05	15·33	15·60	15·87	16·15	16·43	16·71	16·98	17·27	17·55	17·84	18·13	5
15·54	15·83	16·10	16·38	16·67	16·97	17·25	17·53	17·82	18·12	18·43	18·71	160
16·02	16·32	16·60	16·90	17·19	17·50	17·79	18·08	18·38	18·68	19·00	19·30	5
16·51	16·82	17·11	17·41	17·71	18·03	18·33	18·63	18·94	19·25	19·58	19·88	170
16·99	17·31	17·61	17·90	18·23	18·57	18·87	19·18	19·50	19·81	20·15	20·47	5
17·48	17·81	18·12	18·44	18·76	19·11	19·40	19·72	20·05	20·39	20·73	21·05	180
17·96	18·30	18·62	18·95	19·28	19·65	19·94	20·27	20·61	20·95	21·36	21·64	5
18·45	18·80	19·13	19·47	19·80	20·18	20·48	20·82	21·17	21·52	21·88	22·22	190
18·93	19·29	19·63	19·98	20·32	20·72	21·02	21·37	21·77	22·08	22·45	22·81	5
19·43	19·79	20·14	20·49	20·85	21·20	21·56	21·92	22·29	22·65	23·03	23·39	200
19·91	20·28	20·64	21·01	21·27	21·73	22·10	22·46	22·84	23·21	23·60	23·97	5
20·40	20·78	21·14	21·52	21·89	22·26	22·63	23·01	23·40	23·78	24·18	24·56	210
20·88	21·27	21·64	22·03	22·41	22·79	23·17	23·56	23·95	24·34	24·75	25·14	5
21·37	21·77	22·15	22·54	22·93	23·32	23·71	24·10	24·51	24·91	25·33	25·73	220
21·85	22·26	22·65	23·06	23·45	23·85	24·25	24·65	25·06	25·47	25·90	26·31	5
22·34	22·76	23·15	23·57	23·97	24·38	24·79	25·20	25·63	26·04	26·48	26·90	230
22·82	23·25	23·65	24·09	24·49	24·91	25·33	25·75	26·19	26·60	27·05	27·48	5
23·31	23·74	24·16	24·60	25·02	25·44	25·86	26·29	26·74	27·17	27·63	28·07	240
23·79	24·23	24·66	25·11	25·54	25·97	26·40	26·84	27·29	27·73	28·20	28·65	5
24·28	24·72	25·16	25·61	26·06	26·50	26·94	27·39	27·85	28·31	28·78	29·24	250
24·76	25·21	25·66	26·12	26·58	27·03	27·48	27·94	28·40	28·87	29·35	29·82	5
25·25	25·71	26·17	26·64	27·10	27·56	28·02	28·48	28·97	29·44	29·93	30·41	260
25·73	26·20	26·67	27·15	27·62	28·09	28·56	29·02	29·53	30·00	30·50	30·99	5
26·23	26·70	27·18	27·66	28·14	28·62	29·10	29·57	30·08	30·57	31·08	31·58	270
26·71	27·19	27·68	28·18	28·66	29·15	29·64	30·12	36·64	31·13	31·65	32·16	5
27·20	27·69	28·19	28·69	29·19	29·68	30·18	30·66	31·19	31·70	32·23	32·75	280
27·68	28·18	28·69	29·20	29·71	30·21	30·72	31·21	31·75	32·26	32·80	33·33	5
28·17	28·68	29·20	29·71	30·23	30·74	31·26	31·76	32·31	32·83	33·38	33·92	290
28·65	29·18	29·70	30·22	30·75	31·27	31·80	32·31	32·88	33·40	33·95	34·50	5
29·15	29·68	30·21	30·73	31·27	31·79	32·34	32·88	33·43	33·97	34·54	35·08	300
71° 50′ 18° 10′	71° 40′ 18° 20′	71° 30′ 18° 30′	71° 20′ 18° 40′	71° 10′ 18° 50′	71° 0′ 19° 0′	70° 50′ 19° 10′	70° 40′ 19° 20′	70° 30′ 19° 30′	70° 20′ 19° 40′	70° 10′ 19° 50′	70° 0′ 20° 0′	

IV.—Tables for the

	69° 50′	69° 40′	69° 30′	69° 20′	69° 10′	69° 0′	68° 50′	68° 40′	68° 30′	68° 20′	68° 10′	68° 0′
	20° 10′	20° 20′	20° 30′	20° 40′	20° 50′	21° 0′	21° 10′	21° 20′	21° 30′	21° 40′	21° 50′	22° 0′
30	3·56	3·62	3·68	3·74	3·79	3·85	3·91	3·97	4·03	4·09	4·15	4·21
2	3·90	3·86	3·92	3·99	4·04	4·11	4·17	4·23	4·30	4·36	4·43	4·49
4	4·14	4·10	4·17	4·24	4·29	4·37	4·43	4·50	4·56	4·63	4·70	4·77
6	4·37	4·34	4·41	4·49	4·54	4·63	4·69	4·76	4·83	4·91	4·98	5·05
8	4·60	4·58	4·66	4·74	4·80	4·89	4·95	5·03	5·09	5·18	5·26	5·33
40	4·84	4·72	4·91	4·99	5·05	5·14	5·21	5·30	5·36	5·45	5·53	5·61
2	5·08	4·96	5·15	5·24	5·30	5·40	5·47	5·56	5·62	5·72	5·81	5·89
4	5·32	5·20	5·40	5·49	5·55	5·66	5·73	5·83	5·89	5·99	6·08	6·17
6	5·56	5·44	5·64	5·74	5·80	5·91	5·99	6·09	6·16	6·27	6·36	6·45
8	5·80	5·79	5·89	5·99	6·06	6·17	6·25	6·35	6·43	6·54	6·63	6·73
50	5·94	6·04	6·13	6·22	6·32	6·42	6·51	6·61	6·71	6·81	6·91	7·01
2	6·18	6·28	6·38	6·47	6·57	6·68	6·77	6·87	6·97	7·08	7·18	7·29
4	6·42	6·52	6·62	6·72	6·82	6·93	7·03	7·14	7·24	7·36	7·46	7·57
6	6·65	6·76	6·87	6·97	7·07	7·19	7·29	7·40	7·51	7·63	7·74	7·85
8	6·89	7·00	7·11	7·22	7·33	7·44	7·55	7·67	7·78	7·90	8·02	8·13
60	7·13	7·24	7·36	7·47	7·58	7·70	7·81	7·93	8·05	8·17	8·30	8·41
2	7·37	7·48	7·60	7·72	7·83	7·96	8·07	8·20	8·32	8·45	8·57	8·69
4	7·61	7·72	7·85	7·97	8·08	8·21	8·33	8·46	8·58	8·72	8·85	8·97
6	7·84	7·96	8·09	8·22	8·33	8·47	8·59	8·73	8·85	8·99	9·12	9·25
8	8·08	8·20	8·34	8·47	8·59	8·72	8·85	8·99	9·12	9·27	9·40	9·53
70	8·31	8·44	8·58	8·71	8·85	8·98	9·12	9·26	9·40	9·54	9·68	9·82
2	8·55	8·68	8·83	8·96	9·10	9·23	9·38	9·52	9·67	9·81	9·95	10·10
4	8·79	8·92	9·07	9·21	9·35	9·49	9·64	9·79	9·93	10·09	10·23	10·38
6	9·02	9·16	9·32	9·46	9·60	9·75	9·90	10·05	10·20	10·37	10·50	10·66
8	9·26	9·40	9·56	9·71	9·86	10·00	10·16	10·32	10·47	10·64	10·78	10·94
80	9·50	9·64	9·81	9·96	10·11	10·26	10·42	10·58	10·75	10·91	11·05	11·22
2	9·74	9·88	10·05	10·21	10·36	10·51	10·68	10·85	11·02	11·19	11·33	11·50
4	9·97	10·12	10·30	10·46	10·61	10·77	10·94	11·11	11·28	11·46	11·61	11·78
6	10·21	10·36	10·54	10·71	10·87	11·02	11·20	11·38	11·55	11·73	11·89	12·06
8	10·45	10·61	10·79	10·96	11·13	11·28	11·46	11·64	11·82	12·00	12·17	12·34
90	10·70	10·86	11·03	11·21	11·39	11·56	11·73	11·91	12·09	12·27	12·45	12·63
2	10·93	11·10	11·28	11·47	11·64	11·81	11·99	12·17	12·36	12·54	12·72	12·91
4	11·17	11·34	11·52	11·72	11·89	12·07	12·25	12·44	12·62	12·81	13·00	13·19
6	11·41	11·58	11·77	11·97	12·14	12·32	12·51	12·70	12·89	13·10	13·28	13·47
8	11·65	11·82	12·02	12·21	12·39	12·58	12·76	12·97	13·15	13·37	13·55	13·75
100	11·88	12·07	12·26	12·45	12·65	12·84	13·03	13·23	13·43	13·63	13·83	14·03
2	12·11	12·31	12·51	12·71	12·90	13·10	13·29	13·50	13·70	13·90	14·10	14·31
4	12·34	12·55	12·75	12·96	13·15	13·35	13·55	13·76	13·96	14·17	14·38	14·59
6	12·58	12·79	13·00	13·20	13·40	13·61	13·81	14·03	14·23	14·45	14·65	14·87
8	12·82	13·03	13·24	13·45	13·65	13·86	14·07	14·29	14·50	14·72	14·93	15·15
110	13·06	13·27	13·49	13·70	13·91	14·12	14·33	14·55	14·77	14·99	15·21	15·43
	69° 50′	69° 40′	69° 30′	69° 20′	69° 10′	69° 0′	68° 50′	68° 40′	68° 30′	68° 20′	68° 10′	68° 0′
	20° 10′	20° 20′	20° 30′	20° 40′	20° 50′	21° 0′	21° 10′	21° 20′	21° 30′	21° 40′	21° 50′	22° 0′

Reduction of Distances.

67° 50′	67° 40′	67° 30′	67° 20′	67° 10′	67° 0′	66° 50′	66° 40′	66° 30′	66° 20′	66° 10′	66° 0′	
22° 10′	22° 20′	22° 30′	22° 40′	22° 50′	23° 0′	23° 10′	23° 20′	23° 30′	23° 40′	23° 50′	24° 0′	
4·27	4·33	4·39	4·46	4·52	4·58	4·64	4·71	4·77	4·83	4·90	4·96	30
4·55	4·62	4·68	4·76	4·82	4·88	4·95	5·02	5·09	5·15	5·22	5·29	2
4·84	4·91	4·97	5·06	5·12	5·19	5·26	5·33	5·41	5·47	5·55	5·62	4
5·12	5·20	5·27	5·35	5·42	5·49	5·57	5·65	5·73	5·80	5·87	5·95	6
5·41	5·48	5·56	5·65	5·72	5·80	5·88	5·96	6·04	6·12	6·20	6·28	8
5·69	5·77	5·85	5·94	6·02	6·10	6·19	6·27	6·36	6·44	6·52	6·61	40
5·98	6·06	6·14	6·24	6·32	6·41	6·50	6·59	6·68	6·76	6·85	6·94	2
6·26	6·35	6·44	6·53	6·62	6·71	6·81	6·90	7·00	7·09	7·18	7·27	4
6·54	6·64	6·73	6·82	6·92	7·02	7·12	7·21	7·31	7·41	7·50	7·60	6
6·83	6·93	7·02	7·12	7·22	7·32	7·43	7·52	7·63	7·73	7·83	7·93	8
7·12	7·22	7·32	7·42	7·53	7·63	7·74	7·84	7·95	8·05	8·16	8·27	50
7·40	7·50	7·61	7·71	7·83	7·93	8·05	8·15	8·27	8·38	8·48	8·60	2
7·69	7·79	7·90	8·00	8·13	8·23	8·36	8·46	8·58	8·70	8·81	8·93	4
7·97	8·08	8·20	8·30	8·43	8·54	8·67	8·78	8·90	9·02	9·14	9·26	6
8·26	8·36	8·49	8·60	8·73	8·85	8·98	9·09	9·22	9·34	9·47	9·59	8
8·54	8·65	8·79	8·91	9·03	9·16	9·29	9·41	9·54	9·66	9·80	9·92	60
8·83	8·94	9·08	9·20	9·33	9·46	9·60	9·72	9·85	9·99	10·13	10·25	2
9·11	9·23	9·37	9·50	9·63	9·77	9·91	10·03	10·17	10·31	10·45	10·58	4
9·40	9·52	9·67	9·79	9·93	10·07	10·22	10·35	10·49	10·63	10·78	10·91	6
9·68	9·81	9·96	10·09	10·23	10·37	10·53	10·66	10·81	10·95	11·10	11·24	8
9·97	10·11	10·25	10·39	10·54	10·68	10·83	10·98	11·13	11·28	11·42	11·58	70
10·25	10·39	10·55	10·68	10·84	10·98	11·14	11·29	11·45	11·60	11·75	11·91	2
10·54	10·68	10·84	10·98	11·14	11·29	11·45	11·60	11·76	11·92	12·08	12·24	4
10·82	10·97	11·13	11·28	11·44	11·60	11·76	11·92	12·08	12·24	12·41	12·57	6
11·11	11·26	11·43	11·58	11·74	11·91	12·07	12·23	12·40	12·56	12·74	12·90	8
11·39	11·54	11·72	11·88	12·04	12·22	12·38	12·55	12·72	12·89	13·07	13·24	80
11·68	11·83	12·01	12·17	12·34	12·52	12·69	12·86	13·04	13·21	13·39	13·57	2
11·96	12·12	12·31	12·47	12·64	12·83	13·00	13·18	13·36	13·53	13·72	13·90	4
12·25	12·41	12·60	12·77	12·94	13·13	13·31	13·49	13·68	13·85	14·05	14·23	6
12·53	12·70	12·89	13·07	13·24	13·44	13·62	13·81	14·00	14·17	14·38	14·56	8
12·82	13·00	13·19	13·37	13·55	13·74	13·93	14·12	14·31	14·50	14·70	14·89	90
13·10	13·29	13·48	13·66	13·85	14·05	14·24	14·43	14·63	14·82	15·03	15·22	2
13·39	13·58	13·77	13·96	14·15	14·35	14·55	14·75	14·95	15·14	15·35	15·55	4
13·67	13·87	14·07	14·26	14·45	14·66	14·86	15·06	15·26	15·46	15·68	15·88	6
13·95	14·16	14·36	14·55	14·75	14·96	15·17	15·37	15·58	15·78	16·00	16·21	8
14·24	14·45	14·65	14·85	15·06	15·27	15·48	15·69	15·90	16·11	16·33	16·54	100
14·52	14·73	14·95	15·15	15·36	15·57	15·79	16·00	16·22	16·43	16·65	16·87	2
14·81	15·02	15·24	15·44	15·66	15·88	16·10	16·31	16·53	16·75	16·98	17·20	4
15·09	15·31	15·53	15·74	15·96	16·18	16·41	16·63	16·84	17·07	17·31	17·53	6
15·38	15·60	15·83	16·03	16·26	16·49	16·72	16·94	17·16	17·39	17·63	17·86	8
15·66	16·88	16·11	16·33	16·56	16·80	17·03	17·26	17·49	17·72	17·96	18·19	110
67° 50′	67° 40′	67° 30′	67° 20′	67° 10′	67° 0′	66° 50′	66° 40′	66° 30′	66° 20′	66° 10′	66° 0′	
22° 10′	22° 20′	22° 30′	22° 40′	22° 50′	23° 0′	23° 10′	23° 20′	23° 30′	23° 40′	23° 50′	24° 0′	

IV. (*cont.*)—Tables for the

	69° 50′ 20° 10′	69° 40′ 20° 20′	69° 30′ 20° 30′	69° 20′ 20° 40′	69° 10′ 20° 50′	69° 0′ 21° 0′	68° 50′ 21° 10′	68° 40′ 21° 20′	68° 30′ 21° 30′	68° 20′ 21° 40′	68° 10′ 21° 50′	68° 0′ 22° 0′
112	13·29	13·52	13·73	13·95	14·16	14·37	14·59	14·82	15·04	15·26	15·49	15·71
4	13·53	13·76	13·98	14·20	14·41	14·63	14·85	15·08	15·30	15·53	15·76	15·99
6	13·77	14·00	14·22	14·45	14·67	14·88	15·11	15·35	15·57	15·80	16·03	16·27
8	14·01	14·24	14·47	14·70	14·92	15·14	15·37	15·61	15·84	16·07	16·31	16·55
120	14·25	14·48	14·71	14·94	15·18	15·40	15·64	15·87	16·11	16·35	16·59	16·83
5	14·85	15·08	15·32	15·57	15·81	16·02	16·29	16·53	16·78	17·03	17·28	17·53
130	15·44	15·68	15·94	16·19	16·44	16·66	16·94	17·19	17·46	17·71	17·98	18·23
5	16·04	16·29	16·55	16·81	17·05	17·30	17·59	17·85	18·13	18·39	18·67	18·93
140	16·63	16·89	17·17	17·44	17·71	17·95	18·24	18·51	18·80	19·07	19·36	19·64
5	17·23	17·50	17·78	18·06	18·34	18·60	18·89	19·18	19·47	19·76	20·05	20·34
150	17·82	18·11	18·40	18·68	18·97	19·26	19·55	19·85	20·15	20·45	20·74	21·05
5	18·42	18·71	19·01	19·30	19·60	19·90	20·20	20·51	20·82	21·13	21·43	21·75
160	19·01	19·31	19·62	19·92	20·24	20·54	20·85	21·17	21·49	21·81	22·13	22·45
5	19·61	19·92	20·23	20·54	20·87	21·19	21·50	21·83	22·16	22·49	22·82	23·15
170	20·20	20·52	20·85	21·17	21·50	21·83	22·16	22·49	22·83	23·17	23·51	23·85
5	20·80	21·13	21·46	21·79	22·13	22·47	22·71	23·15	23·50	23·85	24·20	24·55
180	21·39	21·73	22·17	22·41	22·77	23·11	23·46	23·81	24·17	24·53	24·89	25·25
5	21·99	22·34	22·78	23·03	23·40	23·76	24·11	24·47	24·84	25·21	25·58	25·95
190	22·58	22·94	23·40	23·65	24·03	24·40	24·76	25·13	25·52	25·89	26·18	26·65
5	23·18	23·55	24·01	24·28	24·66	25·04	25·41	25·79	26·29	26·58	26·87	27·35
200	23·77	24·15	24·53	24·90	25·29	25·68	26·07	26·46	26·87	27·27	27·67	28·06
5	24·37	24·75	25·14	25·52	25·92	26·32	26·72	27·12	27·54	27·95	28·36	28·76
210	24·96	25·36	25·76	26·14	26·56	26·97	27·37	27·78	28·21	28·63	29·05	29·46
5	25·56	25·96	26·37	26·76	27·19	27·61	28·02	28·44	28·88	29·31	29·74	30·16
220	26·15	26·57	26·98	27·39	27·82	28·25	28·68	29·10	29·55	29·99	30·43	30·86
5	26·75	27·17	27·59	28·01	28·45	28·89	29·33	29·76	30·22	30·67	31·12	31·56
230	27·33	27·78	28·20	28·63	29·09	29·53	29·98	30·42	30·90	31·35	31·82	32·26
5	27·93	28·38	28·81	29·25	29·72	30·18	30·63	31·08	31·57	32·03	32·51	32·96
240	28·52	28·99	29·43	29·87	30·35	30·82	31·28	31·74	32·24	32·71	33·20	33·66
5	29·12	29·59	30·04	30·50	30·98	31·46	31·93	32·40	32·91	33·39	33·89	34·36
250	29·71	30·18	30·65	31·12	31·62	32·10	32·59	33·07	33·58	34·08	34·58	35·07
5	30·31	30·78	31·26	31·74	32·25	32·74	33·64	33·73	34·25	34·76	35·27	35·77
260	30·90	31·39	31·88	32·36	32·88	33·38	33·89	34·39	34·92	35·44	35·96	36·47
5	31·50	31·99	32·49	32·98	33·51	34·02	34·54	35·05	35·59	36·12	36·65	37·17
270	32·08	32·60	33·10	33·61	34·15	34·67	35·19	35·71	36·27	36·80	37·35	37·88
5	32·68	33·20	33·71	34·23	34·78	35·31	35·84	36·37	36·94	37·48	38·04	38·58
280	33·27	33·84	34·33	34·86	35·42	35·95	36·50	37·03	37·61	38·16	38·73	39·29
5	33·87	34·41	34·94	35·48	36·05	36·60	37·15	37·69	38·28	38·84	39·42	39·98
290	34·46	35·01	35·56	36·10	36·68	37·24	37·80	38·35	38·95	39·52	40·12	40·68
5	35·06	35·61	36·17	36·73	37·31	37·88	38·45	39·02	39·62	40·21	40·81	41·38
300	35·65	36·22	36·79	37·37	37·94	38·52	39·10	39·69	40·30	40·90	41·50	42·10
	69° 50′ 20° 10′	69° 40′ 20° 20′	69° 30′ 20° 30′	69° 20′ 20° 40′	69° 10′ 20° 50′	69° 0′ 21° 0′	68° 50′ 21° 10′	68° 40′ 21° 20′	68° 30′ 21° 30′	68° 20′ 21° 40′	68° 10′ 21° 50′	68° 0′ 22′ 0

Reduction of Distances.

67° 50′ 22° 10′	67° 40′ 22° 20′	67° 30′ 22° 30′	67° 20′ 22° 40′	67° 10′ 22° 50′	67° 0′ 23° 0′	66° 50′ 23° 10′	66° 40′ 23° 20′	66° 30′ 23° 30′	66° 20′ 23° 40′	66° 10′ 23° 50′	66° 0′ 24° 0′	
15·95	16·17	16·41	16·63	16·86	17·10	17·34	17·57	17·81	18·04	18·20	18·52	112
16·23	16·46	16·70	16·92	17·16	17·41	17·65	17·88	18·13	18·36	18·61	18·85	4
16·52	16·75	16·99	17·22	17·46	17·71	17·96	18·20	18·44	18·68	18·94	19·18	6
16·80	17·04	17·28	17·52	17·76	18·01	18·27	18·51	18·76	19·00	19·27	19·51	8
17·09	17·33	17·58	17·82	18·07	18·32	18·58	18·83	19·08	19·33	19·60	19·85	120
17·79	18·05	18·31	18·56	18·82	19·08	19·35	19·61	19·88	20·13	20·42	20·68	5
18·51	18·77	19·04	19·30	19·58	19·84	20·12	20·40	20·67	20·94	21·24	21·51	130
19·22	19·49	19·77	20·05	20·33	20·60	20·89	21·18	21·47	21·74	22·06	22·33	5
19·94	20·21	20·51	20·79	21·08	21·36	21·66	21·97	22·26	22·55	22·87	23·16	140
20·60	20·93	21·24	21·53	21·83	22·13	22·43	22·75	23·06	23·36	23·69	23·99	5
21·36	21·66	21·97	22·28	22·59	22·90	23·21	23·53	23·85	24·17	24·50	24·51	150
22·07	22·38	22·70	23·02	23·34	23·66	23·98	24·32	24·65	24·97	25·32	25·64	5
22·78	23·10	23·44	23·76	24·09	24·42	24·76	25·10	25·44	25·78	26·14	26·47	160
23·49	23·83	24·17	24·50	25·84	25·18	25·53	25·89	26·24	26·58	26·96	27·49	5
24·21	24·55	24·90	25·24	25·60	25·95	26·30	26·67	27·03	27·38	27·77	28·12	170
24·91	25·27	25·63	25·99	26·35	26·71	27·07	27·45	27·83	28·19	28·59	28·94	5
25·63	25·99	26·37	26·73	27·10	27·47	27·85	28·23	28·62	28·99	29·40	29·77	180
26·33	26·72	27·10	27·47	27·85	28·23	28·62	29·02	29·42	29·80	30·22	30·60	5
27·06	27·44	27·83	28·21	28·61	28·99	29·40	29·80	30·21	30·61	31·03	31·42	190
27·76	28·16	28·56	28·96	29·36	29·76	30·17	30·59	31·01	31·41	31·85	32·25	5
28·48	28·88	29·29	29·70	30·12	30·53	30·95	31·37	31·81	32·22	32·66	33·09	200
29·19	29·60	30·02	30·44	30·87	31·29	31·72	32·16	32·61	33·02	33·48	33·91	5
29·89	30·32	30·76	31·18	31·62	32·05	32·49	32·94	33·40	33·83	34·30	34·74	210
30·51	31·04	31·49	31·93	32·37	32·82	33·26	33·73	34·20	34·63	35·12	35·56	5
31·32	31·74	32·22	32·67	33·13	33·58	34·04	34·51	34·99	35·44	35·94	36·39	220
32·04	32·46	32·95	33·41	33·88	34·35	34·81	35·29	35·79	36·24	36·76	37·22	5
32·75	33·19	33·69	34·15	34·63	35·11	35·59	36·08	36·58	37·05	37·57	38·04	230
33·46	33·91	34·42	34·90	35·38	35·87	36·36	36·86	37·38	37·85	38·39	38·87	5
34·16	34·64	35·15	35·64	36·14	36·64	37·13	37·65	38·17	38·66	39·20	39·69	240
34·88	35·37	35·88	36·38	36·89	37·40	37·90	38·43	38·97	39·47	40·02	40·52	5
35·59	36·10	36·62	37·12	37·65	38·16	38·68	39·21	39·76	40·27	40·84	41·36	250
36·30	36·82	37·35	37·87	38·40	38·92	39·45	40·00	40·56	41·08	41·66	42·18	5
37·01	37·54	38·08	38·61	39·15	39·69	40·23	40·78	41·35	41·89	42·47	43·01	260
37·72	38·27	38·81	39·35	39·90	40·45	41·00	41·57	42·15	42·69	43·29	43·83	5
38·43	39·00	39·55	40·09	40·66	41·21	41·78	42·34	42·94	43·50	44·10	44·65	270
39·14	39·72	40·28	40·84	41·41	41·98	42·55	43·13	43·74	44·31	44·92	45·48	5
39·85	40·44	41·01	41·58	42·16	42·74	43·33	43·91	44·53	45·11	45·73	46·30	280
40·56	41·16	41·74	42·32	42·91	43·50	44·10	44·70	45·33	45·92	46·65	47·13	5
41·28	41·88	42·48	43·06	43·67	44·27	44·87	45·48	46·12	46·72	47·36	47·96	290
42·00	42·60	43·21	43·81	44·42	45·03	45·64	46·27	46·92	47·53	48·18	48·80	5
42·71	43·32	43·94	44·56	45·18	45·80	46·43	47·07	47·71	48·34	48·99	49·63	300
67° 50′ 22° 10′	67° 40′ 22° 20′	67° 30′ 22° 30′	67° 20′ 22° 40′	67° 10′ 22° 50′	67° 0′ 23° 0′	66° 50′ 23° 10′	66° 40′ 23° 20′	66° 30′ 23° 30′	66° 20′ 23° 40′	66° 10′ 23° 50′	66° 0′ 24° 0′	

APPENDIX IV.

DIAGRAM FOR THE REDUCTION OF TANGENTS AND DISTANCES.

THE difficulty of calculating tangents and corrected distances has suggested to me the idea of the diagram included in this volume.

At the bottom of the diagram are written the distances from 0 to 250.

In the vertical line on the right side are written the angles and tangents.

The angles are marked every 6 minutes, every degree being divided into ten parts.

In the vertical line on the left side are drawn the angles and the quantities to be subtracted from the readings of the wires in order to have the corrected distances. The use of this diagram is as follows:—

1. *To find the Tangent of an Angle.*—Pull a thread from point 0 on left side to the corresponding angle in the column of angles (right side); this line meets the vertical line of distance at a certain point. From this point follow the horizontal line up to the column of tangents where you will find the value of the tangent.

Example.—Find out tangent of $3^\circ 45'$ with corrected distance 212. The vertical line of 212 meets the line $3^\circ 45'$ at a certain point. Following the horizontal line we see in the column of tangents that the tangent is between 13 and 14 at a certain point which we reckon as very nearly 13·92. The exact tangent is 13·90, and the error 0·02 which may be considered as unimportant.

2. *To find the Corrected Distances.*—Pull a thread from point 0 on the right side to the corresponding angle in the column of angles (left side); this line meets the vertical line of distance at a certain point. From this point follow the horizontal line up to the column of corrected distances, where you find the quantity to be subtracted in order to have the corrected distance.

Example.—

Difference of extreme wires . .	202
Zenithal angle	$14^\circ\ 20'$

Find the corrected distance.

Pull the thread between 0° (right side) and $14^\circ\ 20'$. It meets vertical line 202 at a certain point, and following the horizontal line from this point, we find that the quantity to be subtracted is between 12 and 13 at a certain point which we reckon as very nearly 12·35. The accurate calculation would give us 12·38.

The corrected distance is thus

$$202 - 12 \cdot 35 = 189 \cdot 65.$$

The error of 0·03 is quite unimportant.

APPENDIX V.

VARIOUS TABLES.

We add a table of logarithms of tangents from 0° to 30°, and of numbers from 0 to 2000. These tables are computed with four decimals. They are expeditious and more accurate than a slide rule, and besides that correct enough to calculate the difference of levels even between station pegs.

Example.—

Find tangent of $17^\circ\ 22'$ for corrected distance 372.

$$\begin{array}{lrl} \log \tan 17^\circ\ 22' = & \bar{1}\cdot 4952 & \text{Table II.} \\ \log \quad 372 = & 2\cdot 5705 & \text{Table I.} \\ \hline & 2\cdot 0657 & \end{array}$$

The corresponding number is in Table III. between columns 3 and 4; we easily reckon it as

116·32

An accurate calculation with fine decimals would give

116·34

The difference of 0·02 is unimportant for tacheometrical methods.

TABLE I.—LOGARITHMS OF NUMBERS 0–500.

No.	0	1	2	3	4	5	6	7	8	9
0	..	0000	3010	4771	6021	6990	7782	8451	9031	9542
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4628	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981

TABLE II.—LOGARITHMS OF NUMBERS 500–1000.

No.	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9699	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

TABLE III.—LOGARITHMS OF NUMBERS 1000–1500.

No.	0	1	2	3	4	5	6	7	8	9
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
101	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082
102	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124
103	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166
104	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208
105	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249
106	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290
107	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330
108	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370
109	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410
110	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449
111	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488
112	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527
113	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565
114	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603
115	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641
116	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678
117	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715
118	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752
119	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788
120	0792	0795	0799	0803	0806	0810	0813	0817	0821	0824
121	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860
122	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896
123	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931
124	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966
125	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000
126	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035
127	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069
128	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103
129	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136
130	1139	1143	1146	1149	1153	1156	1159	1163	1166	1169
131	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202
132	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235
133	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268
134	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300
135	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332
136	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364
137	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396
138	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427
139	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458
140	1461	1464	1467	1471	1474	1477	1480	1483	1486	1489
141	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520
142	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550
143	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581
144	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611
145	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641
146	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670
147	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700
148	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729
149	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758

TABLE IV.—LOGARITHMS OF NUMBERS 1500–2000.

No.	0	1	2	3	4	5	6	7	8	9
150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787
151	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816
152	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844
153	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872
154	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901
155	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928
156	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956
157	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984
158	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011
159	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038
160	2041	2044	2047	2049	2052	2055	2057	2060	2063	2066
161	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092
162	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119
163	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146
164	2148	2151	2154	2156	2159	2162	2164	2167	2170	2272
165	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198
166	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225
167	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251
168	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276
169	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302
170	2304	2307	2310	2312	2315	2317	2320	2322	2325	2327
171	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353
172	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378
173	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403
174	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428
175	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453
176	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477
177	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502
178	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526
179	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550
180	2553	2555	2558	2560	2562	2565	2567	2570	2572	2574
181	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598
182	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622
183	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646
184	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669
185	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693
186	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716
187	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739
188	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762
189	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785
190	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808
191	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831
192	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853
193	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876
194	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898
195	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920
196	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942
197	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964
198	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986
199	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008

TABLE I.—LOGARITHMS

	0°	1°	2°	3°	4°
0	—	$\bar{2}$·2419	$\bar{2}$·5431	$\bar{2}$·7194	$\bar{2}$·8446
1	$\bar{4}$·4637	·2491	·5467	·7218	·8465
2	·7648	·2562	·5503	·7242	·8483
3	·9408	·2631	·5538	·7266	·8501
4	$\bar{3}$·0658	·2700	·5573	·7290	·8518
5	·1627	·2767	·5608	·7313	·8536
6	·2419	$\bar{2}$·2833	$\bar{2}$·5643	$\bar{2}$·7337	$\bar{2}$·8554
7	·3088	·2899	·5677	·7360	·8572
8	·3668	·2963	·5711	·7383	·8589
9	·4180	·3026	·5745	·7406	·8607
10	·4637	·3089	·5779	·7429	·8624
11	·5051	·3150	·5812	·7452	·8642
12	$\bar{3}$·5429	$\bar{2}$·3211	$\bar{2}$·5845	$\bar{2}$·7475	$\bar{2}$·8659
13	·5777	·3271	·5879	·7497	·8676
14	·6099	·3330	·5910	·7520	·8693
15	·6398	·3389	·5943	·7542	·8711
16	·6678	·3446	·5975	·7564	·8728
17	·6942	·3503	·6007	·7582	·8745
18	$\bar{3}$·7190	$\bar{2}$·3559	$\bar{2}$·6038	$\bar{2}$·7609	$\bar{2}$·8762
19	·7425	·3614	·6070	·7631	·8778
20	·7648	·3669	·6100	·7652	·8795
21	·7859	·3722	·6132	·7674	·8812
22	·8061	·3776	·6163	·7695	·8829
23	·8255	·3829	·6193	·7717	·8845
24	$\bar{3}$·8439	$\bar{2}$·3881	$\bar{2}$·6223	$\bar{2}$·7739	$\bar{2}$·8862
25	·8617	·3932	·6253	·7760	·8878
26	·8787	·3983	·6283	·7781	·8895
27	·8951	·4033	·6313	·7802	·8911
28	·9109	·4083	·6343	·7823	·8927
29	·9261	·4132	·6372	·7844	·8944

of Tangents.

5°	6°	7°	8°	9°	
$\bar{2}$·9420	$\bar{1}$·0216	$\bar{1}$·0891	$\bar{1}$·1478	$\bar{1}$·1997	0
·9434	·0228	·0902	·1487	·2005	1
·9448	·0240	·0912	·1496	·2013	2
·9463	·0252	·0923	·1505	·2022	3
·9477	·0264	·0933	·1514	·2030	4
·9492	·0277	·0943	·1524	·2038	5
$\bar{2}$·9506	$\bar{1}$·0289	$\bar{1}$·0954	$\bar{1}$·1533	$\bar{1}$·2046	6
·9520	·0300	·0964	·1542	·2054	7
·9534	·0312	·0974	·1551	·2062	8
·9549	·0324	·0984	·1560	·2070	9
·9563	·0336	·0995	·1569	·2078	10
·9577	·0348	·1005	·1578	·2086	11
$\bar{2}$·9591	$\bar{1}$·0360	$\bar{1}$·1015	$\bar{1}$·1587	$\bar{1}$·2094	12
·9605	·0371	·1025	·1596	·2102	13
·9619	·0383	·1035	·1605	·2110	14
·9632	·0395	·1045	·1613	·2118	15
·9646	·0406	·1055	·1622	·2126	16
·9660	·0418	·1066	·1631	·2134	17
$\bar{2}$·9674	$\bar{1}$·0430	$\bar{1}$·1076	$\bar{1}$·1640	$\bar{1}$·2142	18
·9687	·0441	·1086	·1649	·2150	19
·96	·0453	·1096	·1658	·2158	20
·9715	·0464	·1106	·1666	·2166	21
·9728	·0476	·1155	·1675	·2174	22
·9742	·0487	·1125	·1684	·2181	23
$\bar{2}$·9756	$\bar{1}$·0499	$\bar{1}$·1135	$\bar{1}$·1693	$\bar{1}$·2189	24
·9769	·0510	·1145	·1702	·2197	25
·9782	·0521	·1155	·1710	·2205	26
·9796	·0533	·1165	·1719	·2213	27
·9809	·0544	·1175	·1728	·2220	28
·9822	·0555	·1184	·1736	·2228	29

TABLE I.—LOGARITHMS

	0°	1°	2°	3°	4°
30	$\bar{3}$·9409	$\bar{2}$·4181	$\bar{2}$·6401	$\bar{2}$·7865	$\bar{2}$·8960
31	·9551	·4229	·6430	·7885	·8976
32	·9689	·4276	·6458	·7906	·8992
33	·9822	·4323	·6487	·7927	·9008
34	·9952	·4370	·6515	·7947	·9024
35	$\bar{2}$·0078	·4416	·6543	·7967	·9040
36	·0200	$\bar{2}$·4461	$\bar{2}$·6571	$\bar{2}$·7987	$\bar{2}$·9056
37	·0319	·4506	·6599	·8008	·9071
38	·0435	·4551	·6627	·8028	·9087
39	·0548	·4595	·6654	·8048	·9103
40	·0658	·4638	·6682	·8057	·9118
41	·0765	·4682	·6709	·8087	·9134
42	$\bar{2}$·0870	$\bar{2}$·4724	$\bar{2}$·6736	$\bar{2}$·8107	$\bar{2}$·9150
43	·0972	·4767	·6762	·8126	·9165
44	·1072	·4809	·6789	·8146	·9180
45	·1169	·4850	·6815	·8165	·9196
46	·1265	·4892	·6842	·8186	·9211
47	·1358	·4932	·6868	·8204	·9226
48	$\bar{2}$·1450	$\bar{2}$·4973	$\bar{2}$·6894	$\bar{2}$·8223	$\bar{2}$·9241
49	·1539	·5013	·6920	·8242	·9256
50	·1627	·5053	·6945	·8261	·9272
51	·1713	·5092	·6971	·8280	·9287
52	·1797	·5131	·6996	·8299	·9302
53	·1880	·5170	·7021	·8317	$\bar{2}$·9316
54	$\bar{2}$·1962	$\bar{2}$·5208	$\bar{2}$·7046	$\bar{2}$·8336	·9331
55	·2041	·5246	·7071	·8355	·9346
56	·2119	·5283	·7096	·8373	·9361
57	·2196	·5321	·7121	·8392	·9376
58	·2271	·5358	·7145	·8410	·9390
59	·2346	·5394	·7170	·8428	·9405
60	$\bar{2}$·2419	$\bar{2}$·5431	$\bar{2}$·7194	$\bar{2}$·8446	$\bar{2}$·9420

OF TANGENTS—*cont.*

5°	6°	7°	8°	9°	
$\bar{2}$·9836	$\bar{1}$·0567	$\bar{1}$·1194	$\bar{1}$·1745	$\bar{1}$·2236	30
·9849	·0578	·1204	·1754	·2244	31
·9862	·0589	·1214	·1762	·2252	32
·9875	·0600	·1223	·1771	·2259	33
·9888	·0611	·1233	·1779	·2267	34
·9901	·0622	·1243	·1788	·2275	35
$\bar{2}$·9915	$\bar{1}$·0633	$\bar{1}$·1252	$\bar{1}$·1797	$\bar{1}$·2282	36
·9927	·0644	·1262	·1805	·2290	37
·9940	·0656	·1272	·1814	·2298	38
·9953	·0667	·1281	·1822	·2305	39
·9966	·0677	·1291	·1831	·2313	40
·9979	·0688	·1300	·1839	·2321	41
$\bar{2}$·9992	$\bar{1}$·0699	$\bar{1}$·1310	$\bar{1}$·1848	$\bar{1}$·2328	42
$\bar{1}$·0005	·0710	·1319	·1856	·2336	43
·0017	·0721	·1329	·1864	·2343	44
·0030	·0732	·1338	·1873	·2351	45
·0043	·0743	·1348	·1881	·2359	46
·0055	·0754	·1357	·1890	·2366	47
$\bar{1}$·0068	$\bar{1}$·0764	$\bar{1}$·1367	$\bar{1}$·1898	$\bar{1}$·2374	48
·0080	·0775	·1376	·1906	·2381	49
·0093	·0786	·1385	·1915	·2389	50
·0105	·0796	·1395	·1923	·2396	51
·0118	·0807	·1404	·1931	·2404	52
$\bar{1}$·0130	$\bar{1}$·0818	·1413	·1939	·2411	53
·0143	·0828	$\bar{1}$·1423	$\bar{1}$·1948	$\bar{1}$·2419	54
·0155	·0839	·1432	·1956	·2426	55
·0167	·0849	·1441	·1964	·2433	56
·0180	·0860	·1450	·1972	·2441	57
·0192	·0870	·1460	·1981	·2448	58
·0204	·0881	·1469	·1989	·2456	59
$\bar{1}$·0216	$\bar{1}$·0891	$\bar{1}$·1478	$\bar{1}$·1997	$\bar{1}$·2463	60

TABLE II.—LOGARITHMS

	10°	11°	12°	13°	14°
0	$\bar{1}$·2463	$\bar{1}$·2886	$\bar{1}$·3275	$\bar{1}$·3634	$\bar{1}$·3968
1	·2471	·2893	·3281	·3639	·3973
2	·2478	·2900	·3287	·3645	·3978
3	·2485	·2907	·3293	·3651	·3984
4	·2493	·2913	·3299	·3657	·3989
5	·2500	·2920	·3306	·3662	·3994
6	$\bar{1}$·2507	$\bar{1}$·2927	$\bar{1}$·3312	$\bar{1}$·3668	$\bar{1}$·4000
7	·2515	·2934	·3318	·3674	·4005
8	·2522	·2940	·3324	·3679	·4011
9	·2529	·2947	·3330	·3685	·4016
10	·2536	·2953	·3336	·3691	·4021
11	·2544	·2960	·3343	·3697	·4027
12	$\bar{1}$ 2551	$\bar{1}$·2967	$\bar{1}$·3349	$\bar{1}$·3702	$\bar{1}$·4032
13	·2558	·2973	·3355	·3708	·4037
14	·2565	·2980	·3361	·3714	·4042
15	·2573	·2987	·3367	·3719	·4048
16	·2580	·2993	·3373	·3725	·4053
17	·2587	·3000	·3379	·3731	·4058
18	$\bar{1}$·2594	$\bar{1}$·3006	$\bar{1}$·3385	$\bar{1}$·3736	$\bar{1}$·4064
19	·2601	·3013	·3391	·3742	·4069
20	·2609	·3019	·3397	·3748	·4074
21	·2616	·3026	·3403	·3753	·4079
22	·2623	·3033	·3409	·3759	·4085
23	·2630	·3039	·3415	·3764	·4090
24	$\bar{1}$·2637	$\bar{1}$·3046	$\bar{1}$·3422	$\bar{1}$·3770	$\bar{1}$·4095
25	·2644	·3052	·3427	·3776	·4100
26	·2651	·3059	·3434	·3781	·4106
27	·2658	·3065	·3440	·3787	·4111
28	·2665	·3072	·3446	·3792	·4116
29	·2673	·3078	·3452	·3798	·4121

OF TANGENTS.

15°	16°	17°	18°	19°	
$\bar{1}$·4281	$\bar{1}$·4575	$\bar{1}$·4853	$\bar{1}$·5118	1·5370	0
·4286	·4580	·4858	·5122	·5374	1
·4291	·4584	·4862	·5126	·5378	2
·4296	·4589	·4867	·5131	·5382	3
·4301	·4594	·4871	·5135	·5386	4
·4306	·4599	·4876	·5139	·5390	5
$\bar{1}$·4311	$\bar{1}$·4603	$\bar{1}$·4880	$\bar{1}$·5143	$\bar{1}$·5394	6
·4316	·4608	·4885	·5148	·5398	7
·4321	·4613	·4889	·5152	·5402	8
·4326	·4618	·4894	·5156	·5406	9
·4331	·4622	·4898	·5161	·5411	10
·4336	·4627	·4903	·5165	·5415	11
$\bar{1}$·4341	$\bar{1}$·4632	$\bar{1}$·4907	$\bar{1}$·5169	$\bar{1}$·5419	12
·4346	·4637	·4912	·5173	·5423	13
·4351	·4641	·4916	·5178	·5427	14
·4356	·4646	·4921	·5182	·5431	15
·4361	·4651	·4925	·5186	·5435	16
·4366	·4655	·4930	·5190	·5439	17
$\bar{1}$·4371	$\bar{1}$·4660	$\bar{1}$·4934	$\bar{1}$·5195	$\bar{1}$·5443	18
·4376	·4665	·4938	·5199	·5447	19
·4381	·4669	·4943	·5203	·5451	20
·4385	·4674	·4947	·5207	·5455	21
·4391	·4679	·4952	·5211	·5459	22
·4395	·4683	·4956	·5216	·5463	23
$\bar{1}$·4400	$\bar{1}$·4688	$\bar{1}$·4961	$\bar{1}$·5220	$\bar{1}$·5467	24
·4405	·4693	·4965	·5224	·5471	25
·4410	·4697	·4970	·5228	·5475	26
·4415	·4702	·4974	·5233	·5479	27
·4420	·4707	·4978	·5237	·5483	28
·4425	·4711	·4983	·5241	·5487	29

TABLE II.—LOGARITHMS

	10°	11°	12°	13°	14°
30	$\bar{1}$·2680	$\bar{1}$·3085	$\bar{1}$·3458	$\bar{1}$·3804	$\bar{1}$·4127
31	·2687	·3091	·3463	·3809	·4132
32	·2694	·3097	·3469	·3815	·4137
33	·2701	·3104	·3475	·3820	·4142
34	·2708	·3110	·3481	·3826	·4147
35	·2715	·3117	·3487	·3831	·4153
36	$\bar{1}$·2722	$\bar{1}$·3123	$\bar{1}$·3493	$\bar{1}$·3837	$\bar{1}$·4158
37	·2729	·3130	·3499	·3842	·4163
38	·2736	·3136	·3505	·3848	·4168
39	·2743	·3142	·3511	·3853	·4173
40	·2750	·3149	·3517	·3859	·4178
41	·2757	·3155	·3523	·3864	·4184
42	$\bar{1}$·2763	$\bar{1}$·3162	$\bar{1}$·3529	$\bar{1}$·3870	$\bar{1}$·4189
43	·2770	·3168	·3535	·3875	·4194
44	·2777	·3174	·3540	·3881	·4199
45	·2784	·3181	·3546	·3886	·4204
46	·2791	·3187	·3552	·3892	·4209
47	·2798	·3193	·3558	·3897	·4214
48	$\bar{1}$·2805	$\bar{1}$·3200	$\bar{1}$·3564	$\bar{1}$·3903	$\bar{1}$·4220
49	·2812	·3206	·3570	·3908	·4225
50	·2819	·3212	·3576	·3914	·4230
51	·2825	·3218	·3581	·3919	·4235
52	·2832	·3225	·3587	·3924	·4240
53	·2839	·3231	·3593	·3930	·4245
54	$\bar{1}$·2846	$\bar{1}$·3237	$\bar{1}$·3599	$\bar{1}$·3935	$\bar{1}$·4250
55	·2853	·3244	·3605	·3941	·4255
56	·2859	·3250	·3610	·3946	·4260
57	·2866	·3256	·3616	·3951	·4265
58	·2873	·3262	·3622	·3957	·4270
59	·2880	·3268	·3628	·3962	·4275
60	$\bar{1}$·2886	$\bar{1}$·3275	$\bar{1}$·3634	$\bar{1}$·3968	$\bar{1}$·4281

OF TANGENTS—*cont.*

15°	16°	17°	18°	19°	
$\bar{1}$·4430	$\bar{1}$·4716	$\bar{1}$·4987	$\bar{1}$·5245	$\bar{1}$·5491	30
·4435	·4721	·4992	·5249	·5495	31
·4440	·4725	·4996	·5254	·5499	32
·4445	·4730	5000	·5258	·5503	33
·4449	·4735	·5005	·5262	·5507	34
·4454	·4739	·5009	·5266	·5511	35
$\bar{1}$·4459	$\bar{1}$·4744	$\bar{1}$·5014	$\bar{1}$·5270	$\bar{1}$·5516	36
·4464	·4748	·5018	·5274	·5520	37
·4469	·4753	·5022	·5279	·5524	38
·4474	·4758	·5027	·5283	·5528	39
·4479	·4762	·5031	·5287	·5532	40
·4484	·4767	·5035	·5291	·5536	41
$\bar{1}$·4488	$\bar{1}$·4771	$\bar{1}$·5040	$\bar{1}$·5295	$\bar{1}$·5539	42
·4493	·4776	·5044	·5299	·5543	43
·4498	·4781	·5048	·5304	·5547	44
·4503	·4785	·5053	·5308	·5551	45
·4508	·4790	·5057	·5312	·5555	46
·4513	·4794	·5062	·5316	·5559	47
$\bar{1}$·4517	$\bar{1}$·4799	$\bar{1}$·5066	$\bar{1}$·5320	$\bar{1}$·5563	48
·4522	·4803	·5070	·5324	·5567	49
·4527	·4808	·5075	·5328	·5571	50
·4532	·4813	·5079	·5333	·5575	51
·4537	·4817	·5083	·5337	·5579	52
·4541	·4822	·5088	·5341	·5583	53
$\bar{1}$·4546	$\bar{1}$·4826	$\bar{1}$·5092	$\bar{1}$·5345	$\bar{1}$·5587	54
·4551	·4831	·5096	·5349	·5591	55
·4556	·4835	·5100	·5353	·5595	56
·4561	·4840	·5105	·5357	·5599	57
·4565	·4844	·5109	·5361	·5603	58
·4570	·4849	·5113	·5366	·5607	59
$\bar{1}$·4575	$\bar{1}$·4853	$\bar{1}$·5118	$\bar{1}$·5370	$\bar{1}$·5611	60

TABLE III.—LOGARITHMS

	20°	21°	22°	23°	24°
0	$\bar{1}$·5611	$\bar{1}$.5842	$\bar{1}$·6064	$\bar{1}$·6279	$\bar{1}$·6486
1	·5615	·5845	·6068	·6282	·6489
2	·5618	·5849	·6071	·6285	·6493
3	·5622	·5853	·6075	·6289	·6496
4	·5626	·5857	·6079	·6293	·6499
5	·5630	·5861	·6082	·6296	·6503
6	$\bar{1}$·5634	$\bar{1}$·5864	$\bar{1}$·6086	$\bar{1}$·6300	$\bar{1}$·6506
7	·5638	·5868	·6089	·6303	·6510
8	·5642	·5872	·6093	·6307	·6513
9	·5646	·5876	·6097	·6310	·6516
10	·5650	·5879	·6100	·6313	·6520
11	·5654	·5883	·6104	·6317	·6523
12	$\bar{1}$·5658	$\bar{1}$·5887	$\bar{1}$·6108	$\bar{1}$·6321	1·6527
13	·5661	·5891	·6111	·6324	·6530
14	·5665	·5894	·6115	·6327	·6533
15	·5669	·5898	·6118	·6331	·6537
16	·5673	·5902	·6122	·6334	·6540
17	·5677	·5906	·6126	·6338	·6543
18	$\bar{1}$·5681	$\bar{1}$·5909	$\bar{1}$·6129	$\bar{1}$·6341	$\bar{1}$·6547
19	·5685	·5913	·6133	·6345	·6550
20	·5689	·5917	·6136	·6348	·6553
21	·5693	·5920	·6140	·6352	·6557
22	·5696	·5924	·6144	·6355	·6560
23	·5700	·5928	·6147	·6359	·6564
24	$\bar{1}$·5704	$\bar{1}$·5932	$\bar{1}$·6151	1·6362	$\bar{1}$·6567
25	·5708	·5935	·6154	·6366	·6570
26	·5712	·5939	·6158	·6369	·6574
27	·5716	·5943	·6161	·6373	·6577
28	·5720	·5947	·6165	·6376	·6580
29	·5723	·5950	·6169	·6380	·6584

OF TANGENTS.

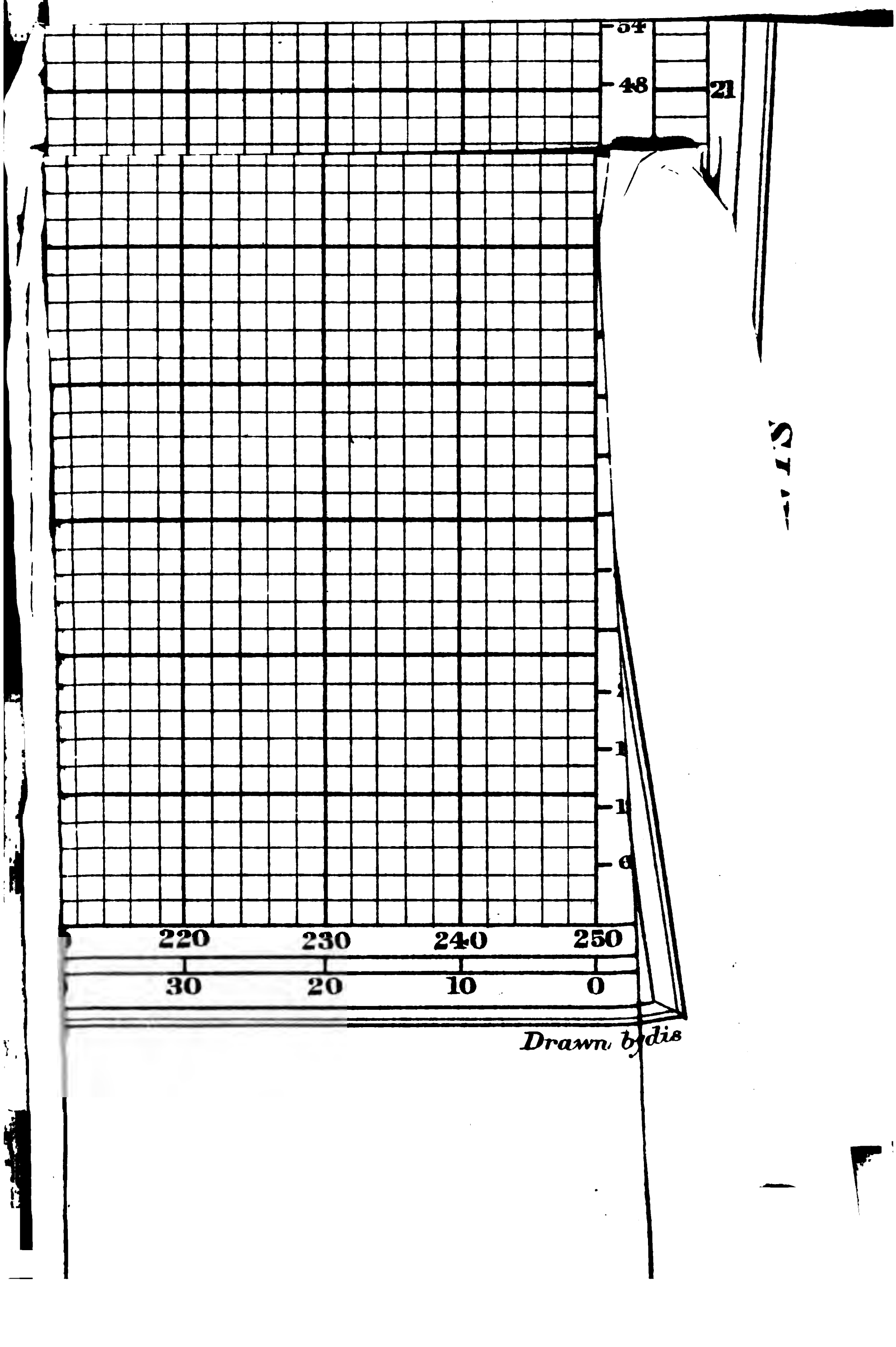
25°	26°	27°	28°	29°	
$\bar{1}$·6687	$\bar{1}$·6882	$\bar{1}$·7072	$\bar{1}$·7257	$\bar{1}$·7438	0
·6690	·6885	·7075	·7260	·7440	1
·6693	·6888	·7078	·7263	·7443	2
·6697	·6891	·7081	·7266	·7446	3
·6700	·6895	·7084	·7269	·7449	4
·6703	·6898	·7087	·7272	·7452	5
$\bar{1}$·6706	$\bar{1}$·6901	$\bar{1}$·7090	$\bar{1}$·7275	$\bar{1}$·7455	6
·6710	·6904	·7093	·7278	·7458	7
·6713	·6907	·7097	·7281	·7461	8
·6716	·6911	·7100	·7284	·7464	9
·6720	·6914	·7103	·7287	·7467	10
·6723	·6917	·7106	·7290	·7470	11
$\bar{1}$·6726	$\bar{1}$·6920	$\bar{1}$·7109	$\bar{1}$·7293	$\bar{1}$·7473	12
·6729	·6923	·7112	·7296	·7476	13
·6733	·6927	·7115	·7299	·7479	14
·6736	·6930	·7118	·7302	·7482	15
·6739	·6933	·7121	·7305	·7485	16
·6743	·6936	·7125	·7308	·7488	17
$\bar{1}$·6746	$\bar{1}$·6939	$\bar{1}$·7128	$\bar{1}$·7311	$\bar{1}$·7491	18
·6749	·6942	·7131	·7314	·7494	19
·6752	·6946	·7134	·7317	·7497	20
·6756	·6949	·7137	·7320	·7500	21
·6759	·6952	·7140	7323	·7503	22
·6762	·6955	·7143	·7326	·7506	23
$\bar{1}$·6765	$\bar{1}$·6958	$\bar{1}$·7146	$\bar{1}$·7330	$\bar{1}$·7509	24
·6769	·6961	·7149	·7333	·7512	25
·6772	·6965	·7152	·7336	·7515	26
·6775	·6968	·7155	·7339	·7518	27
·6778	·6971	·7159	·7342	·7520	28
·6782	·6974	·7162	·7345	·7523	29

TABLE III.—LOGARITHMS

	20°	21°	22°	23°	24°
30	$\bar{1}$·5727	$\bar{1}$·5954	$\bar{1}$·6172	$\bar{1}$·6383	$\bar{1}$·6587
31	·5731	·5958	·6176	·6386	·6590
32	·5735	·5961	·6179	·6390	·6569
33	·5739	·5965	·6183	·6393	·6597
34	·5743	·5969	·6186	·6397	·6600
35	·5747	·5972	·6190	·6400	·6604
36	$\bar{1}$·5750	$\bar{1}$·5976	$\bar{1}$·6194	$\bar{1}$·6404	$\bar{1}$·6607
37	·5754	·5980	·6197	·6407	·6610
38	·5758	·5983	·6201	·6411	·6614
39	·5762	·5987	·6204	·6414	·6617
40	·5766	·5991	·6208	·6417	·6620
41	·5770	·5995	·6211	·6421	·6624
42	$\bar{1}$·5773	$\bar{1}$·5998	$\bar{1}$·6215	$\bar{1}$·6424	$\bar{1}$·6627
43	·5777	·6002	·6218	·6428	·6630
44	·5781	·6006	·6222	·6431	·6634
45	·5785	·6009	·6226	·6435	·6637
46	·5789	·6013	·6229	·6438	·6640
47	·5792	·6017	·6233	·6441	·6644
48	$\bar{1}$·5796	$\bar{1}$·6020	$\bar{1}$·6236	$\bar{1}$·6445	$\bar{1}$·6647
49	·5800	·6024	·6240	·6448	·6650
50	·5804	·6028	·6243	·6452	·6654
51	·5808	·6031	·6247	·6455	·6657
52	·5811	·6035	·6250	·6459	·6660
53	·5815	·6039	·6254	·6462	·6664
54	$\bar{1}$·5819	$\bar{1}$·6042	$\bar{1}$·6257	$\bar{1}$·6465	$\bar{1}$·6667
55	·5823	·6046	·6261	·6469	·6670
56	·5827	·6049	·6264	·6472	·6673
57	·5830	·6053	·6268	·6476	·6677
58	·5834	6057	·6271	·6479	·6680
59	·5838	·6060	·6275	·6482	·6683
60	$\bar{1}$·5842	$\bar{1}$·6064	$\bar{1}$·6279	$\bar{1}$·6486	$\bar{1}$·6687

OF TANGENTS—*cont.*

25°	26°	27°	28°	29°	
$\bar{1}$·6785	$\bar{1}$·6977	$\bar{1}$·7165	$\bar{1}$·7348	$\bar{1}$·7526	**30**
·6788	·6980	·7168	·7351	·7529	**31**
·6791	·6984	·7171	·7354	·7532	**32**
·6795	·6987	·7174	·7357	·7535	**33**
·6798	·6990	·7177	·7360	·7538	**34**
·6801	·6993	·7180	·7363	·7541	**35**
$\bar{1}$·6804	$\bar{1}$·6996	$\bar{1}$·7183	$\bar{1}$·7366	$\bar{1}$·7544	**36**
·6808	·6999	·7186	·7369	·7547	**37**
·6811	·7003	·7189	·7372	·7550	**38**
·6814	·7006	·7192	·7375	·7553	**39**
·6817	·7009	·7195	·7378	·7556	**40**
·6821	·7012	·7199	·7381	·7559	**41**
$\bar{1}$·6824	$\bar{1}$·7015	$\bar{1}$·7202	$\bar{1}$·7384	$\bar{1}$·7562	**42**
·6827	·7018	·7205	·7387	·7565	**43**
·6830	·7021	·7208	·7390	·7568	**44**
·6834	·7025	·7211	·7393	·7570	**45**
·6837	·7028	·7214	·7396	·7573	**46**
·6840	·7031	·7217	·7399	·7576	**47**
$\bar{1}$·6843	$\bar{1}$·7034	$\bar{1}$·7220	$\bar{1}$·7402	$\bar{1}$·7579	**48**
·6846	·7037	·7223	·7405	·7582	**49**
·6850	·7040	·7226	·7408	·7585	**50**
·6853	·7043	·7229	·7411	·7588	**51**
·6856	·7047	·7232	·7414	·7591	**52**
·6859	·7050	·7235	·7417	·7594	**53**
$\bar{1}$·6863	$\bar{1}$·7053	$\bar{1}$·7238	$\bar{1}$·7420	$\bar{1}$·7597	**54**
·6866	·7056	·7241	·7423	·7600	**55**
·6869	·7059	·7244	·7426	·7603	**56**
·6872	·7062	·7248	·7429	·7606	**57**
·6875	·7065	·7251	·7432	·7609	**58**
·6879	·7068	·7254	·7435	·7612	**59**
$\bar{1}$·6882	$\bar{1}$·7072	$\bar{1}$·7257	$\bar{1}$·7438	$\bar{1}$·7614	**60**


48
21
220
230
240
250
30
20
10
0
Drawn by dis

ook may be kept